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THE MELTING POINTS OF RHODIUM AND IRIDIUM, AND THE USE OF THE NERNST GLOWER IN HIGH TEMPERATURE MEASUREMENTS.

By C. E. Mendenhall and L. R. Ingersoll.

IN the following paper we shall consider the use of the Nernst glower for the optical measurement of certain high temperatures, and discuss the result of our determinations, by this method, of the melting points of rhodium and iridium, and the temperature of normal brilliancy of the glower, as well as the melting point of its material and its radiation constants.

As is well known there are two general radiation methods which have proved satisfactory for temperature measurements — one based on the variation of total radiation with temperature, *i. e.*, $J = \sigma T^4$; the other on the variation with temperature of the emission of a small group of wave-lengths, that is on the isochromatic radiation curve, $\log E_\lambda = K_1 + K_2/T$. As usually used they are based on the theory and radiation laws of a "black body" and can only be correctly applied to determine the temperature of such a body. In the case of non-black bodies such as the Nernst glower since there is no theoretical basis for any radiation law the measurement of high temperatures, even more than in the case of black bodies, depends on the exterpolation of an empirical law. The method of isochromatic observations at some wave-length of the visible spectrum is much simpler to carry out than the total radiation one, and also various considerations (see *sources of error*) indicate that the isochromatic radiation formula is the safest to exterpolate. Our work, therefore,

depending as it does on the determination of the various temperatures of the Nernst glower, is based on the exterpolation of the isochromatic

$$\log E_\lambda = K_1 + K_2/T$$

where E_λ is the emission of the radiation of wave-length λ at the absolute temperature T, and K_1 and K_2 are constants. To establish a temperature scale on this basis evidently requires observations of the values of E_λ — measured in any units whatever — at only two known temperatures, for which purpose we use the melting points of gold and platinum. Knowing this, the temperature corresponding to any other emission may be very easily computed. Substantially the same method has been applied by Waidner and Burgess[1] to the platinum strip of a Joly meldometer.

Arrangement oe Apparatus.

The arrangement of apparatus is shown in Fig. 1. The glower

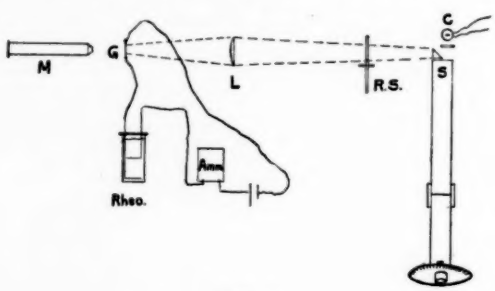

Fig. 1.

G was of the ordinary 110 volt D.C. type furnished by the Nernst Lamp Company of Pittsburgh, and was used with 200 and sometimes even with 500 volts and a large ballast resistance, for in this way only could it be run steadily at the lower temperatures. A milammeter and continuously variable resistance (electrolytic) made possible its perfect control. It was mounted with possibility of adjustment and placed so that it might be observed with the microscope M from one side, while from the other an image was thrown by the lens L on the slit S of a Martens-König spectrophotometer. The aperture of the lens and size of image were so chosen as always

[1] Waidner and Burgess, Bull. Bureau of Standards, 1, p. 189, 1904.

to much more than cover the slit, and more than fill the aperture of the collimating lens of the spectrophotometer. This being so, the brightness of the corresponding half-field in the spectrophotometer was independent of small changes in the position of the glower.

As the comparison source C, a 2 c.p. incandescent lamp behind a ground glass screen was used, the current through it being maintained constant. To increase the accuracy of reading at the higher temperatures, where the ratio of the intensity of the glower to that of the comparison source became large, a rotating sector RS was introduced between the lens and the slit. The sectors were made to reduce approximately in the ratios 1/5, 1/10, and 1/25, the exact values being determined by measurements with a circular dividing engine. The glower was usually protected by a metal shield and cover, to increase the constancy and uniformity of its temperature.

Method of Observation.

The method of observation was in brief to place on top of the glower a very minute particle of the metal to be tested, gradually increase the glower current until the melting point was reached, and then make a number of settings of the spectrophotometer at that temperature, the wave-length used in almost all cases being $\lambda = .606\ \mu$. In some cases, notably gold and platinum, the observer at the microscope could so control the glower current as to maintain the globule of metal part melted and part solid, for some time, during which spectrophotometer settings could be made. In other cases it was found better to determine the current corresponding to the melting temperature by a number of trials, and then to maintain this steady current while the photometer was being read. In cases where it was impossible to maintain this half-melted half-solid condition, it was the melting point and not the point of solidification which was taken, as the latter is subject, in small particles, to great effects of undercooling. It was also the practice not to take the "first melt" of the particles, but subsequent meltings after the globule was in good contact with the glower surface. In practically all cases standardizing observations were made with each glower upon the melting points of gold and platinum, though, as can be seen from Table I. and Fig. 2, the slope of the isochromatic

— determined by log E_{platinum} — log E_{gold} — was remarkably constant for all glowers; that is, K_2 varied but little for the different glowers. The measurement in this way of the emission of the glower in terms of a constant comparison lamp was all that was necessary to determine the temperature scale; and the observations were best handled graphically by simply plotting log E_{gold} against $\frac{1}{T_g + 273}$, and log E_{platinum} against $\frac{1}{T_p + 273}$ where E_{gold} and E_{platinum} are the glower emissions in terms of the comparison lamp. The reciprocal of the temperature corresponding to any other emission E could then be read off at once from the straight line connecting the standard points.

STANDARDIZING TEMPERATURES.

The melting point of gold was taken at 1065° C. which is probably correct to about a degree. The melting point of platinum is however still quite uncertain, and as practically all very high temperature measurements are based in some way on the melting point of platinum, all such temperature determinations above say 1400° C. are subject to corrections of from 10° to over 100°, depending on the value accepted for platinum. The observations of Waidner and Burgess,[1] and of Nernst and v. Wartenburg[2] place the platinum melting point at about 1745° C., while Holborn and Valentiner[3] in their recent work put in at 1789° C. It being at present impossible to decide finally between these two, we give in our tables of results two values for all the temperatures determined, calculated on the respective bases of 1745° C. and 1789° C. for the platinum melting point.

SOURCES OF ERROR.

1. *The Use of Paschen's Isochromatic Equation.* — Since nearly all our measurements involve an exterpolation, it is important to consider carefully the choice of the radiation function which is to be exterpolated.

The work of Paschen[4] shows that up to temperatures of about

[1] Waidner and Burgess, Paper before Am. Phys. Soc., Feb., 1907.

[2] Nernst and v. Wartenburg, Verh. d. Deutsch. Phys. Ges., 1906, pp. 48 and 146.

[3] Holborn and Valentiner, Ann. d. Phys., 22, p. 1, 1907.

[4] F. Paschen, Wied. Ann., 58, p. 455, 1896; 60, p. 662, 1897.

1400° C., or as far as he could test it, and for wave-lengths such that $\lambda T < 3{,}000$, an equation of the form of Wien's

$$E_{(\lambda, T)} = C_1 \lambda^{-a} e^{-c_2/\lambda T} \tag{1}$$

represents with very considerable accuracy the form of the energy curve for various kinds of radiating surfaces; that the isochromatic radiation is well represented by

$$\log E_{(\lambda, T)} = K_1 + K_2/T \tag{2}$$

and that the total radiation from these surfaces follows quite closely the law

$$J = \sigma T^{a-1} \tag{3}$$

where $a > 6$ ($a = 5$ for a black body). As they have no theoretical basis except for a black body, and as such measurements as we are concerned with involve very considerable exterpolations beyond the range of experimental test, it is well to consider briefly the probable validity of these empirical laws. Since the total radiation of a non-black body can only approach as a limit that of a black body, law (3) cannot be expected to hold indefinitely with a constant $a > 5$; a must then diminish towards 5 for high temperatures, and the greater the initial value of a, the greater the final change. Equation (2) is of course a necessary consequence of (1), but (2) *might* be true and (1) not; (3) is a necessary consequence of (1), and if (3) does not hold with constant a, (1) cannot. But a might vary considerably (enough to reconcile (3) with the facts) and still give an approximately linear isochromatic, for (1) may be written

$$\log E_{(\lambda, T)} = \log C_1 - a \log \lambda - \frac{C_2}{\lambda T} \log e$$

which would, for values of λ near unity (1 μ), — since C_1 has been found to be for those radiating surfaces so far studied, between 500,000 and 2,000,000,[1] give an isochromatic almost linear in log E and $1/T$, even although a varied with T.[2]

[1] F. Paschen, loc. cit.

[2] In this connection we have made some attempts to measure a for the glower to see if it is a noticeable function of the temperature. These will be referred to later.

Equations (1) and (2) have recently been discussed by Burgess in a somewhat different manner in connection with the observations of Waidner and Burgess on the radiation of a platinum strip; and these observations also confirm the validity of the linear isochromatic equation (2).

It seems to us then, from these considerations, that the isochromatic radiation formula is, for non-black bodies, the safest to use in exterpolating for high temperature measurements. Two indications of the general correctness of this conclusion are furnished by the results. One is given by the determination of the melting point of palladium which was found to be at 1548° C. or 1576° C. according as the melting point of platinum is assumed at 1745° C. or at 1789° C. Nernst and v. Wartenburg[1] who found the lower value for platinum placed palladium at 1541° C., while Holborn and Valentiner[2] set it at 1582° C., on the other basis for platinum. It will be noted that our values are as much above one as below the other; and while not furnishing in itself a justification for *exterpolating* the isochromatic radiation formula, this does show that it is sensibly a straight line at least between the gold and platinum points. The other point in support of our method depends on measurements made with two different wave-lengths, viz., $\lambda = .606\,\mu$ and $\lambda = .493\,\mu$. The following table shows that temperatures measured with the two different wave-lengths check to within the probable error of measurement, and this could not be expected to be the case unless our equations are at least approximately correct.

Platinum melting point taken at 1789° C.

Glower No. 23, melting point of rhodium	$\lambda = .493\,\mu$	$T = 1967°$ C.
	$\lambda = .606\,\mu$	$T = 1970°$ C.
Glower No. 24, temperature of normal brilliancy	$\lambda = .493\,\mu$	$T = 2189°$ C.
	$\lambda = .606\,\mu$	$T = 2198°$ C.

2. *Temperature Difference Between the Globule of Metal and the Glower.* — This is dependent on the size of the particle and its contact with the glower and may give rise to an uncertainty, due to the use of too large a piece of metal, or of poor contact with the glower, of as much as 100 degrees or more unless proper precautions are

[1] Nernst and v. Wartenburg, loc. cit.
[2] Holborn and Valentiner, loc. cit.

taken. For example, if a small rider of platinum foil only .007 inch thick is hung on the glower, its apparent melting point may be as much as 200 degrees too high, and even a very small piece of metal when first put on the glower and in poor contact, may require a glower temperature of perhaps 50 degrees or more above its melting point before it will melt. This source of error was studied by using globules of various sizes, and guarded against by taking observations only on globules so small (.05 mm.) that the

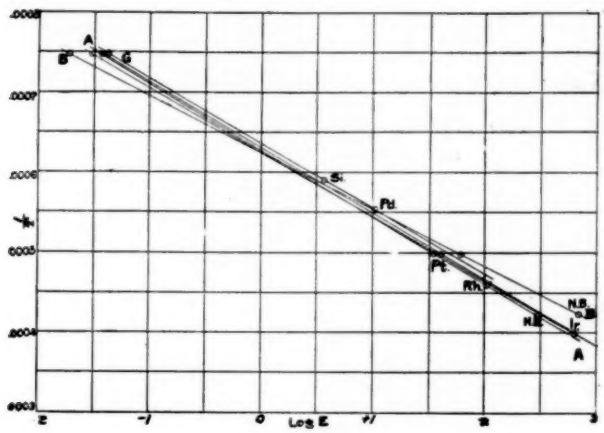

Fig. 2.

melting point was observed to be practically independent of the size. Furthermore the method of measurement tended to eliminate the effect of any small outstanding difference of temperature still remaining; for about the same sized globules of the various metals, gold, platinum, rhodium, iridium, etc., were used, and if this difference can be supposed roughly proportional to the temperature it can be easily seen that it can have little or no effect on the determination of the melting point.

3. *Alloying of the Metals.* — The results are given only for those metals which draw into a clear metallic bead on the surface of the glower: the consistency of readings for successive melts, and for old and fresh globules, renders any appreciable contamination by the material of the glower very improbable. Only in rare instances, and then only when the glower had been used with various metals,

TABLE I.

Melting Points and Temperatures of the Nernst Glower in Degrees C.

Glower No.	2	3	4	5	6	7	8	9	18	22	23	24	34	35	37	38	39	40	41	Mean.
Melting point of Silicon. Pt = 1745			1450	1436	1425	1447	1426	1424												1434
Melting point of Silicon. Pt = 1789			1468	1454	1443	1465	1439	1442												1452
Melting point of Palladium. Pt = 1745	1548	1544	1553	1550	1543															1548
Melting point of Palladium. Pt = 1789	1576	1572	1581	1578	1571															1576
Number of observations.	1	2	1	3	1															
Melting point of Rhodium. Pt = 1745										1899	1908	1902	1906	1902				1917	1912	1907
Melting point of Rhodium. Pt = 1789										1961	1970	1964	1967	1963				1977	1972	1968
Number and quality of observations.										2 good	2 good	1 good	1 fair	1 good				1 good	1 good	
Melting Point of Iridium. Pt = 1745															2288	2303	2279	2307		2292
Melting Point of Iridium. Pt = 1789															2384	2397	2375	2403		2388
Number and quality of observations.															4 good	1 fair	1 good	1 good		

TABLE I.—*Continued.*

Melting Points and Temperatures of the Nernst Glower in Degrees C.

Glower No.	2	3	4	5	6	7	8	9	18	22	23	24	34	35	37	38	39	40	41	Mean.
Temp. of Normal Brill. of Glower. Pt = 1745		2113		2127			2154		2074	1934	2158	2117								2124
Temp. of Normal Brill. of Glower. Pt = 1789		2194		2203			2234		2154	1997	2240	2198								2204
Remarks.		with shield		with shield			with shield		no shield	large glower	no shield	no shield								
Melting point of Glower Material. Pt = 1745							2363											2393		
Melting point of Glower Material. Pt = 1789							2473											2500		
Log $E_{plat.}$—log E_{gold}					3.070						2.954	3.023					3.085		3.013	
C_2 Pt = 1745					17000	(* Second value is obtained by using $\lambda = .493$)					16350 / 16050 *	16700 / 16000					17100		16700	16800
C_2 Pt = 1789					16300						15700 / 15300 *	16100 / 15350					1640		16000	16200

TABLE II.

Summary of the Principal Determinations of the Following Points.

Substance.	Melting Point. °C.	Method.	Melting Point Accepted for Platinum. °C.	Observer.
Silicon.	1430			Tone.
	1434 } ± 8	Optical.	1745	M. and I.
	1452 }	"	1789	"
Palladium.	1541	Wanner pyrometer.	1745	Nernst and v. Wartenburg.
	1582	Thermoelectric.	1789	Holborn and Valentiner.
	1548 } ± 5	Optical.	1745	M. and I.
	1576 }	"	1789	"
Rhodium.	1907 } ± 5	"	1745	"
	1968 }	"	1789	"
Iridium.	2200–2300			Weyde (1879).
	1950			Violle (1879).
	2292 } ± 10	"	1745	M. and I.
	2388 }	"	1789	"
Temperature of glower at normal brilliancy.	2200–2450 abs.[1]	Max. of energy curve.	1745	Lummer and Pringsheim.
	2360 abs.[1]	Efficiency.	1745	Ingersoll.
	1800 abs.	Thermoelectric.		Hartman.
	2400 abs.	Optical.	1745	M. and I.
	2480 abs.	"	1789	"
Melting point of magnesium oxid.	1910	Wanner pyrometer.		Goodwin and Mailey.
	1980–2023	Optical.	1745	M. and I.
Melting point of glower material.	2380	"	1745	"

have inconsistencies arisen which would call for alloying as an explanation. Globules of different metals on the same glower are of course kept as far apart as possible.

4. *Oxidation.* — Observations have been made only in air so far, and only on those metals which oxidize little if at all. Oxidation can usually be readily detected with the microscope.

[1] These values were based on the old values of the black body radiation constants, which have subsequently given by exterpolation the value of 1745 for the platinum melting point.

5. *Purity of the Metals.* — For standardizing points, Heræus platinum and some pure gold kindly given us by Professor L. Kahlenberg, were used. The palladium also came from him. Many specimens of rhodium, ruthenium and iridium were tried, for most of which we are indebted to Professor V. Lenher: they were all, so far as could be learned, prepared by Schuchardt, but no self-consistent results could be obtained with them. That is to say, different samples of supposedly the same metal gave widely different melting points, and different particles of even the same sample would vary considerably. Finally some specimens of iridium, rhodium and ruthenium were obtained from W. C. Heræus, which were furnished as pure, save that the iridium might contain traces of ruthenium. These gave very uniform and consistent readings, and the results here given refer entirely to these specimens. Observations with ruthenium have been so far unsatisfactory on account of its tendency to flux with the glower, in which respect it differs entirely from iridium and rhodium.

Discussion of Results.

The results of our observations are summarized in Tables I. and II. and may be discussed under separate heads, in the order of ascending temperatures. As metals to be studied we have chosen those for whose melting points little if any data could be found, limited by the fact that a number of such metals either oxidize or flux with the material of the glower at the melting temperature. Some of these may be studied by a modified method later.

1. The melting point of silicon, 1434° C. or 1452° C. (according to the value accepted for platinum). It is very easy to observe because of marked surface changes on solidification but the separate determinations show rather large variations. One of the samples used was furnished by Mr. Watts, of the electrochemical department, and supposed to be 99 per cent. pure. Tone,[1] working with silicon which analysis showed to be 95.3 per cent. pure, found the melting point at 1430° C.

2. The melting point of palladium, 1548° C. or 1576° C. was determined as a check on the method. As already noted these values form practically a mean of the best previous determinations.

[1] T. J. Tone, Trans. Am. Electrochem. Soc., 7, p. 245, 1905.

3. The melting point of rhodium, 1907° C. or 1968° C. is such a sharp and easily determined point that it should prove useful as a reference point in high temperature measurements.

4. Temperature of normal brilliancy of the glower. For an .8 amp. 110 volt glower 1.04 mm. in diameter, this may be perhaps placed at 2120° C. or 2200° C., and for one 1.30 mm. in diameter, some 200 degrees lower. This is of course not a very definite point, but is of interest, partly on account of its bearing on the question of luminous efficiency, and partly in connection with recent discrepant estimates of the "temperature of the glower." Determinations have been made by (*a*) Lummer and Pringsheim [1] from measurements of the wave-length of maximum energy, applying Wien's law and assuming that the glower had radiating properties between those of a black body and platinum; they give the limits 2200° and 2450° absolute: (*b*) by one of us,[2] from measurements of the luminous efficiency of the glower, assuming that it is a black body; this gives 2360° absolute: (*c*) by Hartman,[3] using thermocouples of various sizes and an exterpolation method of E. L. Nichols [4] to eliminate the effect of conduction of the wires. He concludes that the temperature is only about 1800° absolute.

The first two of the above agree fairly well with our present determinations, which may be stated in round numbers at 2400° to 2475° absolute for the smaller size of glower now in general use. Unfortunately, without making efficiency measurements and temperature measurements on the same glower and under the same conditions of current and voltage, it is impossible to draw any definite conclusions as to the presence of selective emission which would make the glower more efficient at a given temperature than a black body. However the fair agreement of our determinations with that of Ingersoll indicates that such selective emission, if present at all, is small.

That Hartman's result of less than 1600° C. is widely in error can be readily shown by putting a small bit of platinum on the glower and noting that it melts at a temperature far below that of

[1] Lummer and Pringsheim, Verh. phys. Ges. Berlin, 1, p. 215, 1899.
[2] L. R. Ingersoll, PHYS. REV., 17, p. 371, 1903.
[3] L. W. Hartman, PHYS. REV., 22, p. 351, 1906.
[4] E. L. Nichols, PHYS. REV., 10, p. 234, 1900.

normal brilliancy. But if it is in the form of a fine wire or strip hung over the glower so that part of it is comparatively cool, it cannot be melted at all, or at least only with the greatest difficulty. Herein probably lies the explanation of Hartman's low results, for it would seem that Nichol's exterpolation method does not by any means compensate for the cooling effect of the leads, at least where poor thermal conductivity of the heated body and high temperatures are concerned, so that great temperature gradients are possible. In support of this idea may be mentioned results obtained by this method on the temperature of the candle flame.[1] They place it at less than 1675° C., while it is well known that a fine Wollaston wire may be readily melted in it. To remove the objection that the Wollaston wire may be an alloy of platinum, we used Heræus platinum foil and found that very fine strips could be fused in almost any part of the flame.

5. Melting point of iridium, 2292° C. or 2388° C. To obtain this temperature easily it was desirable to keep the ends of the glower from becoming overheated by hanging strips of platinum on them. Under these circumstances the iridium could be melted into a globule and this remelted some two or three times before it had entirely sublimed. A glower current of 1.3 to 1.5 amp. was required and a 1/25 sector was introduced, as the radiation at the wave-length used was roughly 20,000 times greater at this temperature than at the gold melting point.

6. Melting point of the material of the glower, about 2380° C. or 2490° C. This had to be obtained by slowly raising the temperature of the glower, keeping the photometer set for balance, until fusing occurred, which would be at perhaps 1.8 amp. The values are only an indication, probably too low if anything, of the maximum temperatures attainable with glowers of ordinary composition; by using pure zirconium oxide the limit could probably be raised somewhat. In this connection we have determined roughly the melting points of several specimens of "C.P." magnesium oxide — one from Schuchardt, one from Eimer and Amend, and one furnished us by Mr. Watts. The melting points range from 1980° to 2023° C. (Pt at 1745°), somewhat higher than that ob-

[1] E. L. Nichols, loc. cit., p. 248.

tained by Goodwin and Mailey,[1] viz., 1910° C. It is of interest to note that both these figures are far below the usual estimate, based on experience, of those who have used magnesium oxide practically.

Changes of Radiating Power of the Metals at the Melting Point.

There are a number of very interesting phenomena exhibited by small metallic particles on a Nernst glower, some of which we believe to have escaped notice heretofore — as for instance, the translation, or "walking" of certain particles of metal from one end of the glower to the other. These will be described at length in a forthcoming paper, but it will not be out of place to mention here the surface changes which occur in the metals studied, at their melting points. Thus a surface change so slight as possibly to escape notice in the ordinary process of observing the melting point could be readily detected as a change in the relative brightness of the globule and the glower. Gold shows a marked increase of radiating power on melting; palladium, platinum, rhodium and iridium show the same to a smaller extent, while silicon shows the reverse effect very markedly. With rhodium a second change was also observed at about 1100° C. consisting of a very marked increase of radiating power as the temperature fell — the reverse of the effect at the melt-point — a slight rise in temperature being sufficient to reverse the effect.

The Radiation Constants of the Nernst Glower Material.

If we assume that a law of the form of Wien's applies to the glower, we can interpret the slope constant of the isochromatic as $K_2 = \log e \cdot \frac{C_2}{\lambda}$, from which C_2 can be determined. This gives as a mean value $C_2 = 16{,}500$, which by comparison with the values for a black body and for polished platinum indicates that the glower material is less black even than platinum, as far as visible radiations are concerned.

If we furthermore suppose that a law of the form of Stefan's $J = \sigma T^{a-1}$ holds, it is of interest to determine a, especially to see

[1] Goodwin and Mailey, Trans. Am. Electrochem. Soc., 9, p. 93, 1905.

whether it is a function of T. This we have attempted to do in two ways, but with unsatisfactory results. First, by comparing the monochromatic intensity of radiation of the glower with that of platinum for two known wave-lengths, and at several known temperatures. Such a comparison gives at each temperature a value of a in terms of a for platinum — but it is quite indirect. Second, by measuring the total intensity of radiation of the glower at various temperatures. The results can only be considered to give as a first approximation a value for a of about 9 — higher than for any radiator studied before. They gave no consistent evidence of a variation of a with T.

Summary.

By a graphical exterpolation of the empirical isochromatic radiation formula we have determined the working temperature, and also the maximum attainable temperature of the Nernst glower and obtained at least approximate values for two of its radiation "constants." The glower has also been used as a species of optical meldometer, which, for the determination of high temperature melting points, seems to have certain decided advantages, namely, simplicity of method, great temperature range available, and, at least for certain classes of metals, freedom from contamination and alloying of the metals used. We have applied this method to the determination of a number of melting points, most important of which are those of rhodium at 1907° C. or 1968° C. according to the value accepted for the platinum melting point, and iridium at 2292° C. or 2388° C.

Physical Laboratory,
University of Wisconsin,
April, 1907.

MERCUROUS SULPHATE, CADMIUM SULPHATE AND THE CADMIUM CELL.

BY G. A. HULETT.

THE previous work[1] on standard cells developed a number of facts which indicated that there was some interaction between the cadmium sulphate solution and the mercurous sulphate of the depolarizer, and now considerable experimental evidence on this question has been obtained. From the standpoint of the electrolytic dissociation theory we might expect mercurous sulphate to be less soluble in a cadmium sulphate solution than in water, since the two salts have a common ion. But the solubility was found to be about five times as great in a saturated cadmium sulphate solution as it was in water, and this fact suggested that the common ion might be ineffective in preventing hydrolysis of the mercurous sulphate. The solubility determinations indicated the presence of about a gram of mercury in a liter of the saturated solution but the solubility varied with the time the system had been rotated. This mercurous sulphate, which had been rotated with the cadmium sulphate solution and mercury, gave abnormal results when used as the depolarizer of cells. The attempt to form mercurous sulphate directly in the cadmium sulphate solution by the electrolytic method also gave an abnormal product. These results were only to be understood after more information had been gained about the conditions of equilibrium of the system (mercurous sulphate, mercury, cadmium sulphate and solution). Accordingly an apparatus was devised by which it was possible to rotate this system at a constant temperature and at any desired speed. By means of an adjustable anode of cadmium amalgam, cadmium sulphate and solution, it was possible to determine any change in the E.M.F. between the mercury and the solution of the rotated system, and thus it was possible to follow the changes of concentration of mercury in the solution.

[1] Zeit. Phys. Chem., 49, 483, 1904. PHYS. REV., 22, 321 (and 23, 166).

On rotating the cathode system of the cadmium cell, it was found that the E.M.F. increased steadily for over two weeks and was only fairly constant on continued rotation. The experiment was repeated three times with fresh materials and, while the results varied somewhat, the maximum increase was about .00140 V. After the system had been rotated for thirty days it was allowed to rest but the E.M.F. did not remain constant, it decreased 0.00007 V. the first six hours, 0.00013 in twenty-four hours and then more slowly as time passed. The system was therefore not in equilibrium.

The mercurous sulphate from this rotated system was used as the depolarizer of two cadmium cells. For cell A49 the mercurous sulphate was first brought into a Gooch crucible and washed with a little fresh cadmium sulphate solution, while for cell A50 the depolarizer direct from the rotated tube was used. These cells were set up June 11, 1906, and have since been maintained at 25°. The basis of all E.M.F. values given in this article are Clark cells with electrolytic mercurous sulphate as depolarizer. Their value as determined by absolute measurements is 1.42040 V. at 25°.

Cell.	June 12.	June 14.	June 17.	June 23.	June 26.	July 7.
A49	1.01863	1.01875	1.01868	1.01856	1.01855	1.01847
A50	64	64	64	60	55	47

Cell.	August 14.	October 22.	December 19.	January 3.	February 26.
A49	1.01839	1.01827	1.01823	1.01817	1.01814
A50	38	29	26	20	15

The high preliminary value of these cells and the subsequent decrease is characteristic of cells which contain basic mercurous sulphate in the depolarizer. The cells showed no tendency to stop at the normal value, 1.01840. The results show that the rotated system did not come to equilibrium even when rotated at constant temperature for a month, and unless the changes were due to the dissolved air or to some mechanical change in the materials of the system, they clearly indicate that slow chemical changes were taking place. It seemed possible that the oxygen of the air, dissolved in the cadmium sulphate solution, might have oxidized the mercurous sulphate to the more soluble mercuric compound and,

if the rate of this oxidation was more rapid than the subsequent rate of reduction by the mercury, the observed results could be readily explained. This possibility had been considered[1] and reasons given for rejecting it, but in view of the importance of the results obtained by rotating the cathode systems of standard cells, it has seemed advisable to further test the possibility of an effect due to the dissolved air. The test was carried out with apparatus similar to that used in the previous experiment. An adjustable cell (Fig. 1) was constructed from the materials which were to be tested in the rotating tubes. This cell was kept at 25° and, although the anode leg was frequently removed to test other systems, the cell has not varied over .00001 V. from its value of 1.01838 V. The rotating tubes are represented by Fig. 2.

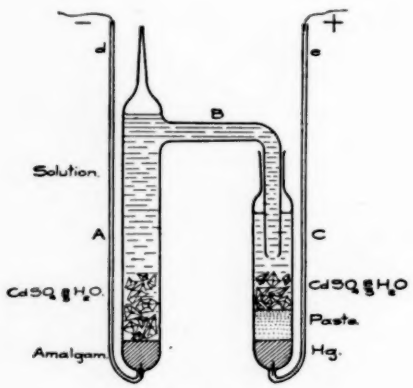

Fig. 1.

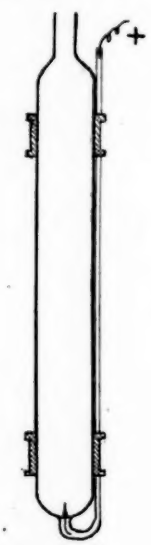

Fig. 2.

the platinum contact wire was sealed through the bottom of the tube, extended 8 mm. up into the tube where it was covered with a thin sheath of glass except for 2 mm. of the end which was exposed and thoroughly amalgamated. A small glass tube was passed over the end of the platinum wire which was outside of the tube, and then sealed to the large glass tube. This little glass tube

[1] PHYS. REV., 23, 180.

was bent as indicated in the figure and served to protect and insulate the copper lead wire. Contact between the platinum and copper wires was made with a little mercury and the rest of the little tube was filled with wax. The rotating tubes used were 20 mm. in diameter and 200 mm. long, each contained 7 c.cm. of mercury, 4 c.cm. of mercurous sulphate, 4 c.cm. of powdered cadmium sulphate and about 50 c.cm. of the saturated solution. These two systems were alike in all respects except that the air was rigorously excluded from tube No II. while tube No I. contained air and was frequently opened.

These tubes with their contents were rotated about their longer axis in the 25° bath just as described[2] for the first experiments. The temperature of 25° was determined by certified 1/50° thermometers and the large 100 liter thermostat seldom varied .02° from this temperature. Care was exercised to keep the bath covered so as to protect the rotating system from the action of light since the black product, formed by the action of light on mercurous sulphate, increases the mercury concentration of the solution.

The materials used were distilled mercury,[1] recrystallized cadmium sulphate (the clear crystals only were used), and a saturated solution of these crystals. The mercurous sulphate was prepared chemically by adding an acid solution of mercurous nitrate to a 1 to 6 sulphuric acid solution. The sulphuric acid contained mercury and was rapidly stirred while the mercurous nitrate solution was added drop by drop. In the first experiments the gray electrolytic mercurous sulphate was used (l. c.) while here the chemically prepared sample was white, distinctly crystalline and coarse-grained. The preparation was brought onto a Gooch crucible, washed with sulphuric acid ($D=1.15$), with absolute alcohol to remove the acid, and finally with the saturated cadmium sulphate solution. Particular attention was given to these washings, because it had been found that sulphuric acid prevented the changes under consideration. The washed mercurous sulphate was mixed with an equal bulk of powdered cadmium sulphate crystals and brought into the rotating tube No. I., which already contained the mercury and

[1] PHYS. REV., 23, 177.
[2] PHYS. REV., 21, 387.

saturated solution. This tube was closed with a cork and a rubber cap and was rotated in the bath.

Tube No. II. was first charged with the mercury and powdered cadmium sulphate and then evacuated and filled with pure hydrogen gas. The tube was evacuated and refilled with hydrogen several times in order to make sure that all the air had been removed from the tube and its contents. The saturated cadmium sulphate solution was brought into a vacuum and shaken until the dissolved air had been practically removed. Hydrogen gas was then allowed to bubble through the solution for three days. The mercurous sulphate was washed as before except the final washings were made with the hydrogen saturated solution and care was taken to avoid drawing air through the sulphate. This sulphate was washed into tube No. II. with some of the hydrogen saturated cadmium sulphate solution, a current of hydrogen prevented the entrance of air and after the tube had been nearly filled with the solution it was evacuated and sealed off before the blow pipe. The materials of this tube No. II. may have retained some hydrogen but they were free from oxygen and when the tube was opened, after two weeks' rotation, the vacuum was still in evidence so there could have been no leak.

The tubes were rotated together and at the same rate. After 33 hours, the rotation was stopped, tube No. I. was brought to an upright position and measured with the anode leg of the adjustable cell.

3	minutes after stopping the rotation the E.M.F. was	1.01960
6	" " " " " " " "	51
13	" " " " " " " "	40
45	" " " " " " " "	30
60	" " " " " " " "	29

The rotation was then continued and after 4 days the observations were,

2	minutes after stopping the rotation the E.M.F. was	1.02036
5	" " " " " " " "	17
10	" " " " " " " "	08
20	hours " " " " " " "	1.01986
60	" " " " " " " "	77
6	days " " " " " " "	

The rotation was then continued for 6 days making a total of ten days and tube No. I. was again tested.

2	minutes	after	stopping	the	rotation	the	E.M.F.	was	.....	1.02035
8	"	"	"	"	"	"	"	"	.....	18
45	"	"	"	"	"	"	"	"	.....	09
2	hours	"	"	"	"	"	"	"	.....	05
6	"	"	"	"	"	"	"	"	.....	00
Ajustable cell.....									.....	1.01838

Evidently the system had about reached its maximum value and was nearly .00200 V. higher than the adjustable cell which had been made from the same materials. The increase in E.M.F. was somewhat greater than in the first experiments but the maximum value obtained has no real significance since it is not an equilibrium point; otherwise there would have been no change after the rotation was stopped. The significance of the decreasing E.M.F. will be considered later, here attention may well be called to some features of the experiment. The adjustable cell was compared with the standards before and after each series of measurements on a rotated system and since there was no variation of the adjustable cell greater than .00001 V., it is certain that the anode tester remained constant and that the observed changes were all in the rotated system. Furthermore, after the solids had settled in the rotated tube, some of the clear supernatant solution was removed, and tested on a clean mercury electrode with the adjustable anode and the value 1.02013 V. was observed. We may therefore be quite confident that the large values obtained for the E.M.F. were due solely to an increase in the mercury concentration of the rotated solution.

Tube No. I. had about reached its maximum value but was again rotated and two days later when both tubes had been rotated a total of twelve days, the hermetically sealed tube No. II. was also tested with the anode leg of the adjustable cell.

The agreement between the two tubes is as good as between different measurements on tube No. I., and since the systems were not in equilibrium the agreement in maximum values is better than was to be expected. The decreasing values depend for one thing on diffusion and this rate of decrease varied at different times for the same system as may be seen from the various measurements on tube No. I. The results obtained, therefore, give a decisive answer to the question at issue and we may conclude that the dissolved air plays no rôle in these rotation experiments.

Time from Stopping the Rotation.	Tube No. II. Air Free.	Tube No. I. Air Saturated.	Time from Stopping the Rotation.	Tube No. II. Air Free.	Tube No. I. Air Saturated.
1¼ minutes.	1.02040 V.	1.02060	24 hours.	1.01972	1.020000
2 "	23		2 days.	60	1.01988
5 "	11	50	4 "	46	64
10 "	03	42	7 "	34	40
30 "	1.01997	36	14 "	12	16
60 "	91	33	21 "	00	
3 hours.	86	22	30 "	1.01800	
6 "	81	17			

Since the dissolved oxygen did not cause the changes in the cathode system, there must have been some interaction between the materials of the rotated tubes which increased the mercury concentration of the solution. If this action was hydrolysis, as has been suggested, an explanation of the observed changes would be possible. Assuming that hydrolysis did take place it followed that the presence of sulphuric acid, one of the products of hydrolysis, would minimize or prevent the changes. This conclusion was tested by the following experiment:

A saturated solution of cadmium sulphate was made tenth molecular in respect to sulphuric acid and this acid solution was used in constructing an adjustable cell and in filling a rotating tube. Electrolytic mercurous sulphate was washed with alcohol and with the acid cadmium sulphate solution and was then made to a paste in the usual way. The other materials, apparatus and method were the same as for the preceding experiments. The adjustable cell, due to the presence of acid, was lower than the cadmium cell (1.01825 V. at 25°), but quite as constant. The cathode system in the tube was rotated as previously described at 4 r.p.m. and after twelve hours it was measured with the anode leg of the adjustable cell with results as follows:

3	minutes after stopping the rotation the E.M.F. was								.....	1.01812 V.
8	"	"	"	"	"	"	"	"	.....	22
30	"	"	"	"	"	"	"	"	.....	22
60	"	"	"	"	"	"	"	"	.....	22
Adjustable cell.....										24

After two days rotation the following results were obtained:

2	minutes	after	stopping	the	rotation	the E.M.F. was	1.01815
5	"	"	"	"	"	"	20
20	"	"	"	"	"	"	24
10	hours	"	"	"	"	"	24
Adjustable cell							25

The tube was then rotated two weeks:

2	minutes	after	stopping	the	rotation	the E.M.F. was	1.018200
4	hours	"	"	"	"	"	21
12	"	"	"	"	"	"	22
3	days	"	"	"	"	"	25
4	"	"	"	"	"	"	23
Adjustable cell							24

This system — mercurous sulphate, mercury, cadmium sulphate and acid solution of cadmium sulphate — came to equilibrium soon after it was brought together and there was then no further change. The system behaved exactly as did the cathode system of the Clark cell.[1] The effect of the sulphuric acid in eliminating the changes in the rotated system is in agreement with the view that the neutral cadmium sulphate solution hydrolyzed the mercurous sulphate, but this is not by itself a proof of the point.

This experiment, with the previous ones, furnishes more evidence on the question of oxidation by dissolved air. The materials and solution in this last experiment were air saturated and frequently opened to the air but the system came to equilibrium at once with the normal E.M.F. This would hardly have been the case if the dissolved air had been oxidizing the mercurous sulphate. In the rotation experiment with the cathode system of the Clark cell the zinc sulphate solution was air saturated, but did not oxidize the mercurous sulphate because the system showed the normal value and did not change on continued rotation.[2]

It will be seen that the experiment just described also gives some

[1] PHYS. REV., 23, 181.

[2] The work of J. W. McBain (J. Phys. Chem., 5, 621, 1901) on the oxidation of ferrous solutions by free oxygen, may be mentioned in this connection. The author found that the rate of oxidation of ferrous sulphate by dissolved air was proportional to the square of the concentration of the ferrous sulphate proportional to the concentration of the oxygen, and that sulphuric acid had only a slightly retarding influence on the rate. The rate was unexpectedly slow, it was shown that precautions prescribed in volumetric analysis for preparing a ferrous sulphate solution from iron wire are quite unnecessary, since with air bubbling through the solution only .05 per cent. of ferric iron was formed

evidence on the question of the size of particles changing the solubility of the mercurous sulphate. A fine powder has an abnormally great solubility but the effect has not been observed until the powder contained a large number of particles with diameters less than one or two microns.[1] Previous experiments have shown that mercurous sulphate which settled rapidly in a 1 to 6 sulphuric acid solution was sufficiently coarse-grained to avoid a measurable effect of the size of the particles on the solubility. But the mercurous sulphate might have been ground to a fine powder in the rotating tubes and this would not only have increased the mercury concentration of the solution, but on standing, the finer more soluble crystals would have disappeared and the mercury concentration would then have decreased. The changes in E.M.F. observed on rotating the cathode system of the cadmium cell corresponded with this explanation, but when a little acid was added the system soon came to equilibrium and the slow changes previously observed were entirely lacking. The same kind of mercurous sulphate and rate of rotation were used in both cases so the observed changes in the first experiment cannot be explained as due to finely divided particles or to grinding of the solids since the presence of the acid in the last experiment could not have effected either of these factors.

The electrolytic mercurous sulphate and the same rate of rotation were used in the experiment with the cathode system of the Clark cell, the system came to equilibrium at once with no changes characteristic of the presence of finely divided particles and the same conclusions may be drawn from that experiment. The method of rotating the tubes[2] was devised in order to avoid grinding the solid contents of the system but at the same time it was necessary to insure a thorough mixing of the cathode materials and this was satisfactorily accomplished by the method and apparatus employed.

THE AGING OF STANDARD CELLS.

In the method of constructing cells devised by Clark,[3] Wright and Thompson[4] and Rayleigh,[5] the mercurous sulphate was washed

[1] Zeit. Phys. Chem., 49, 385, 1901.
[2] PHYS. REV., 23, 177.
[3] Proc. Roy. Soc., 20, 444, 1872.
[4] Phil. Mag., 16, 25, 1833.
[5] Phil. Trans., 175, 411 and 176, 781, 1886.

with water and thus contained the basic salt even if it had not been previously formed in making the mercurous sulphate. Such cells always had a high value (1 to 2 per cent., Rayleigh, l. c., 446) and required a month or two of "aging" before they reached a constant value. On the other hand when the mercurous sulphate was properly prepared and washed so as to exclude the basic salt from the depolarizer,[1] the cells had their constant value at once. Cells have been made with depolarizers which contained known amounts of the basic mercurous sulphate and these cells had a high preliminary value, and then showed the characteristic decrease. Electrolytic mercurous sulphate was shaken with definite amounts of water and an excess of mercury, thus a definite amount of the basic salt was formed and mixed with the undissolved solid. This mixture was made to a paste in the usual way, cell A44 contained one per cent. of the basic salt in the depolarizer, cell A45, three per cent., and cell A46, 10 per cent. while the completely hydrolyzed salt ($2(HgOH).Hg_2SO_4$) was used for cell A41. These cells have been maintained constantly at 25° and their records for over a year are as follows.

Cell.	Jan. 13.	Jan. 26.	Feb. 7.	March 13.	April 13.	May 12.	June 4.
A44	1.01878	1.01871	1.01868	1.01866	1.01865	1.01862	1.01860
A45	64	60	58	55	53	52	47
A46	98	90	86	80	74	69	62
A41	1.01768	760	767	774	776	772	770
Cell.	**July 7.**	**Aug. 14.**	**Nov. 15.**	**Dec. 19.**	**Jan. 3.**	**Feb. 26.**	
A44	1.01857	1.01855	1.01853	1.01854	1.01850	1.01849	
A45	47	47	45	57	46	46	
A46	57	56	44	40	40	44	
A41	764	756	747	741	736	732	

The cells with one per cent. and three per cent. of basic salt have become fairly constant, while cell A46, with 10 per cent. of basic salt in the depolarizer, had the highest preliminary value and has varied the most while the cell with only the basic salt, A41, shows no sign of becoming constant. The depolarizers of cells A35 to A40[2] were made basic by mixing known amounts of the completely hydrolyzed salt with the normal salt. The effect was not as great

[1] PHYS. REV., 23, 172.
[2] PHYS. REV., 22, 328.

as in the experiments just described but of the same general character. The higher values and more rapid rate of decrease were found in the case of the cells with the larger per cent. of basic salt, and these cells were liable to go below the normal value.

On rotating the cathode system of the cadmium cell an increase in the E.M.F. was obtained and when the system was allowed to rest the E.M.F. began to decrease, rapidly at first and then at a decreasing rate. The mercurous sulphate which had been rotated thirty days with the cadmium sulphate solution and mercury was used as the depolarizer of cells A49 and A50 (p. 17), and these cells had a high value and then decreased and are now below normal value and still decreasing. The parallelism between all these facts is evident and indicates that a basic mercurous sulphate was formed in the rotation experiments. The fact that a little sulphuric acid prevented the changes in the cathode system is in agreement with this view and on the whole it seems reasonable to conclude that a neutral cadmium sulphate solution interacts with the depolarizer to form a basic mercurous sulphate. Hydrolysis would account for the facts established by the previous experiments, but the basic salt formed by the action of the cadmium sulphate solution is probably different from the one formed by the action of water, otherwise we would expect the cells with variable amounts of the basic salt (due to water) to show more agreement since it would be a case of two solids saturating a solvent and enough of the basic salt to prevent any action of the cadmium sulphate solution. But whatever the nature of the new phase which formed on rotating the cathode system of the cadmium cell, the subsequent decreasing E.M.F. showed that the system had not attained equilibrium.

There evidently were two oppositely directed changes taking place; the one, assumed to be hydrolysis of the mercurous sulphate, increased the mercury concentration of the solution while the other and much slower change was evidently a secondary action of the mercury on the soluble products of hydrolysis. This secondary action occurred only when the basic salt was or had been present and it caused a decrease of the mercury concentration of the solution. Tube No. I. attained the high value of 1.02060 V. and on standing, decreased to 1.01900 V. in three weeks but on examining the super-

natant solution it was found to have still retained a high value showing 1.02013 V. against a mercury electrode. This test was made by bringing some of the clear liquid onto a mercury electrode in a little tube like the part (*C*) of Fig. 1 and measuring the E.M.F. with the adjustable anode. When left in contact with the mercury electrode the clear liquid showed a slowly decreasing value and shaking it with the mercury seemed to have very little effect in hastening the action.

First reading	1.02013
6 minutes	00
20 "	1.01998
90 "	89
1 day	68
3 days	51
10 "	49

We may therefore conclude that, both in the cells which contained basic salt and in the rotated systems, there was a slow secondary reaction which took place and that this action *was only at the surface of the mercury*. The rate at which the mercury concentration of the solution above the paste decreased depended on the rate of diffusion of the solution through the paste and thus depended on the mechanical fineness of the solid materials of the cathode system. The hydrolysis, or whatever reaction formed the basic salt, could not have attained equilibrium until the secondary action was completed or had formed a new phase. If this view is correct it is probable that the maximum value was not observed in any of the rotation experiments. In the first experiment with the gray electrolytic mercurous sulphate there was a tendency for the mercury to become divided into fine globules, and thus the mercury surface was much increased and consequently the rate of the secondary reaction. In the experiment with the white chemically prepared mercurous sulphate (p. 19) there was no tendency of the mercury to become finely divided and the maximum value was higher than in the first experiment and attained in less time. Evidently variations in the materials and in the apparatus and method used will give quite different results, but the experiments seem to clearly show that in the cathode leg of the cadmium cell we have a system which is in unstable equilibrium.

The changes that have been observed in the E.M.F. of cadmium cells are of course small compared to those obtained in the rotation experiments. The F series of 10 cells which were made February 15, 1904, showed an agreement among themselves of one part in 100,000 for nearly a year, but now they show variations of 5 parts in 100,000 and are from 10 to 15 parts lower than they were when made. The A series of October 21, 1905, showed the same excellent agreement for nearly a year.

Cell.	Oct. 26.	Dec. 14.	March 4.	June 4.	Oct. 2.	Dec. 19.	March 18.
A1	1.01843	1.01843	1.01843	1.01842	1.01842	1.01841	1.01841
A2	42	43	43	42	46	44	43
A3	42	42	43	42			
A4	43	43	43	42	42	42	42
A5	43	43	43	42	41	40	40½
A6	43	43	43	43	42	42	42

The paste in these (A) cells was about 2 cm. deep and the cells have remained undisturbed in the 25° bath, but are not now in as good agreement as formerly, although they have not shown as great a decrease as was exhibited by the F series.

It is probable that cadmium cells, made with a fine-grained depolarizer of good depth, would show less variations than have been shown by the F and A series, since there would be less chance for diffusion and consequently for the changes indicated in this article. A very fine-grained mercurous sulphate retains the sulphuric acid very tenaciously, and the presence of this acid in the cell also tends to retard the changes. But a standard cell which depends on the mechanical conditions of the solid materials and is in unstable equilibrium must be less reproducible and less constant than cells which are free from these defects. In the experiment in which acid was added to the cathode materials of the cadmium cell it was found that the system soon came to equilibrium, and this has suggested that the cadmium cell should have a definite amount of sulphuric acid. A number of these acid cadmium cells have been constructed and are very constant, but as yet insufficient observations have been made on them to judge of their worth.

At the same time the F and A cells were made, Clark cells were also made and with depolarizers made from the same samples of

mercurous sulphate as served for the cadmium cells. These Clark cells are now in good agreement and have the same value as cells made in the last few months from fresh materials. This is in agreement with the results of the rotation experiments on the cathode systems of the Clark and cadmium cells.

The fact that these Clark cells which were made at various times with fresh materials are now in good agreement (± .00002 V.) is proof that they have not changed and that they form a reliable basis of electromotive force. The absolute value of these cells is 1.4204 volts at 25°, as determined by Dr. K. E. Guthe.[1]

The essential points to be observed in constructing these cells have been given.[2] The high temperature coefficient of the Clark cells have made it necessary to have very constant temperature baths. A large copper tank contained 100 liters of kerosene oil and was electrically heated and controlled. A mercury-toluene regulator operated the relay, which in turn controlled the heating current. This regulator consisted of glass tubes joined together in the form of a gridiron so as to be well distributed in the bath. The tubes were filled with toluene, except the outlet, which was filled with mercury. Contact with the relay circuit was made with a very fine platinum wire (.05 mm.), and a condenser was used to prevent sparking. It was found that with the very fine contact wire there was no "fouling" of the mercury and no "drift" of the bath temperature. The heating coils were also in the form of a gridiron, made of glass rods and pianoforte wire, which was wound on the glass rods and then thorougly shellacked. The resistance was 120 ohms and the heating current was only one half ampere (an alternating current was used). The heating surface was large and well distributed in the bath, and only a very small amount of heat was needed, because the bath was well protected with non-conducting materials. The oil was stirred with a motor-driven paddle and there were no differences of temperature in the various parts of the bath that could be detected with a Beckmann thermometer. The temperature was determined by certified 1/50° thermometers, and these thermometers were checked at the transition point of sodium

[1] Bull. Bur. Standards, 2, 33, 1906.

[2] PHYS. REV., 22, 334–338.

sulphate, 32.383°.[1] An Otto Wolff potentiometer was used in the measurements, and its corrections were known and applied when necessary. The galvanometer gave 2 mm. deflection for .00001 V. In making measurements it was found best to join all the anodes of all the cells together, and particular attention was given to the question of insulation. The measurements given are regarded as correct to the .000005 V.

PRINCETON UNIVERSITY,
March 15, 1907.

[1] Richards & Wells, Zeit. Phys. Chem., 43, 471, 1903.

CRYSTAL RECTIFIERS FOR ELECTRIC CURRENTS AND ELECTRIC OSCILLATIONS.

Part I. Carborundum.

By George W. Pierce.

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Introductian. — General H. H. C. Dunwoody of the United States Army has discovered[1] that a crystalline mass of carborundum when supplied with electrodes acts as a receiver for electric waves. In his patents pecification General Dunwoody shows several ways of attaching the electrodes to the crystal. One method is to wind the wires around the two ends of the specimen. Another method is to hold the crystal in a clamp of which the two jaws, insulated from each other, serve as electrodes. The crystal with its electrodes is put into a receiving circuit of a wireless telegraph system with a telephone and battery about the carborundum. The sounds heard in the telephone when a message is being re-

[1] U. S. Patent, No. 837,616, issued December 4, 1906; application filed March 23, 1906.

ceived are like those heard with the electrolytic detector. General Dunwoody also found that the battery could be omitted and the leads of the telephone connected directly about the terminals of the carborundum, and with this arrangement, without the battery, he says that he has read messages from a sending station several hundred miles away.

The present investigation was undertaken in the effort to obtain further knowledge of this interesting property of carborundum. The experiments were extended to include many other crystalline substances, but the present discussion is chiefly confined to carborundum.

Apparatus for Current-voltage Measurements. — Fig. 1 shows a sketch of a form of circuit employed in studying the conductivity of carborundum under various conditions. The crystal of carborundum, held in a clamp, is shown at *Cr*; *B* is a storage battery; *XYZ* is a potentiometer consisting of two fixed plates of zinc *X* and *Z*, and one movable plate *Y*, immersed in a zinc sulphate solution. By means of the voltmeter *V* the difference of potential between the plates *Y* and *Z* could be read, and the resulting current through the carborundum was given by a galvanometer or milliammeter at *A*. The resistance of the galvanometer was so small in comparison with the resistance of the carborundum that the reading of the voltmeter was practically the drop of voltage in the carborundum.

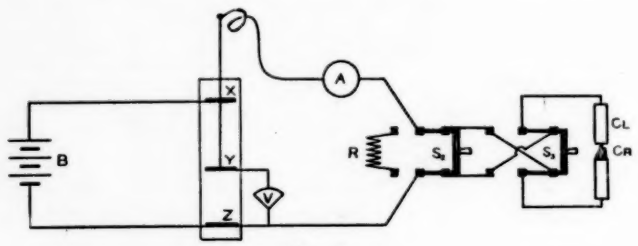

Fig. 1.

The switch S_3 enables the observer to reverse the current in the crystal under examination without reversing the galvanometer. A known resistance at *R*, could be thrown into circuit with the galvanometer for the purpose of calibrating it.

This method of experimenting has previously been employed by Eccles,[1] Guthe and Trowbridge[2] and others in the study of the coherer; by Rothmund and Lessing[3] and by Austin[4] and Armagnat[5] in the study of the electrolytic detector.

Current-voltage Curve for Carborundum. — A curve obtained by plotting the current against voltage in an experiment with carborundum is shown in Fig. 2. It is seen that the current through the carborundum is not a linear function of the voltage about it; the apparent resistance of the substance diminishes with increasing current. Curves of approximately this form were obtained by Greenleaf W. Picard[6] in a study of carborundum. This curve also resembles closely the curves obtained by Rothmund and Lessing, by Austin and by Armagnat for the relation of current to voltage in the electrolytic receiver. It resembles also the building up portion of the current-voltage curve in the coherer as obtained by Eccles.

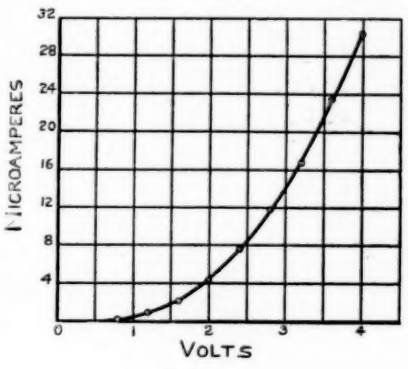

Fig. 2.

Experiments were made by the writer on a great many specimens of carborundum, and curves of approximately the shape shown in Fig. 2 were obtained in all the cases. The value of the current for a given voltage was found to depend on the temperature, and pressure, and on the method of leading the current to the crystal, and was different for different specimens. Before discussing these effects of temperature, pressure, etc., attention is called to a more interesting property of carborundum, the property of unilateral conductivity.

Unilateral Conductivity of Carborundum. — The current through the crystal in one direction under a given electromotive force was

[1] Electrician, 47, pp. 682 and 715, 1901.
[2] PHYS. REV., 11, p. 22, 1900.
[3] Ann. d. Phys., 15, p. 193, 1904.
[4] Bureau of Standards, 2, p. 261, 1906.
[5] Bull. soc. francaise, Session of Apl., 1906, p. 205.
[6] El. World, 48, p. 994, 1906.

found to be different from the current in the opposite direction under the same electromotive force; that is to say, carborundum is unilaterally conductive. This effect may be seen by a reference to

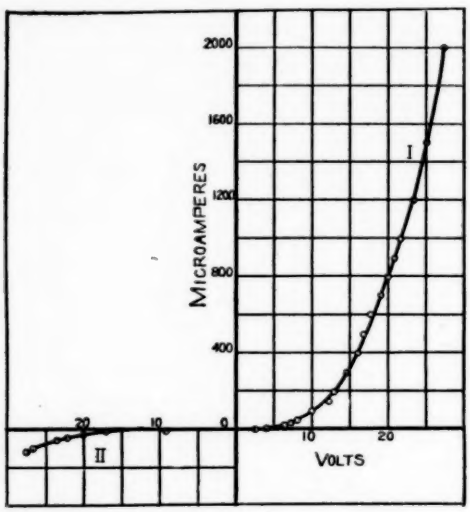

Fig. 3.

Fig. 3. The branch *I* of the curve shows the current, plotted against voltage, when the current is in one direction; branch *II* the corresponding values of the current obtained when the voltage is reversed. The accompanying table, Table I., contains the numerical values from which these curves were plotted. This property of unilateral conductivity of carborundum has been overlooked by previous experimenters on this substance. The property of unilateral conductivity has, however, been previously found in some of the crystalline metallic oxides and sulphides, by Ferdinand Braun. Reference to Braun's work is given in the historical note on page 58. None of the substances investigated by Braun showed such striking asymmetry as that obtained in the present experiments with carborundum.

In the experiment whose result is shown in Fig. 3 and Table I., the specimen of carborundum was held in a clamp under a pressure of about 500 grams, and it is seen that the current in one direction is 100 times as great as the current in the opposite direction when

an electromotive force of 10 volts is applied in the two cases. With increase of current through the specimen, the ratio of the current in the two opposite directions diminishes. At 27.5 volts C_1 is only 17 times C_2.

TABLE I.

Relation of Current to Voltage, Showing unilateral Conductivity of Carborundum.

Volts.	Current in Microamperes.		C_1/C_2.
	C_1, Commutator Left.	C_2, Commutator Right.	
2.2	1		
2.8	2		
4.0	5		
4.7	10		
5.9	20		
6.5	30		
7.3	40		
8.0	50		
10.0	100	1	100
12.1	150		
12.8	200		
14.5	300	5	60
16.0	400		
16.8	500	10	50
17.7	600		
19.4	700		
20.0	800	20	40
21.0	900		
21.9	1,000	30	33
23.2	1,200	50	24
25.0	1,500		
27.5	2,000	120	17

In this particular experiment the piece of carborundum was submerged in an oil bath designed to keep the temperature of the specimen constant. The piece of carborundum was held in a clamp, the jaws of which served to lead the current to the specimen. The oil of which the temperature was 64°C., came freely into contact with the crystal.

Similar effects were obtained at various temperatures between — 10°C. and 100°C., both with and without the use of oil as a bath. A like result was had with different specimens and under different pressures. The relative values of the positive and nega-

tive currents, however, varied from piece to piece, and also was different under different conditions of temperature and pressure. The effects of temperature and pressure are investigated below.

An interesting property of some of the specimens is presented in Fig. 4. The curve of current against voltage obtained when the

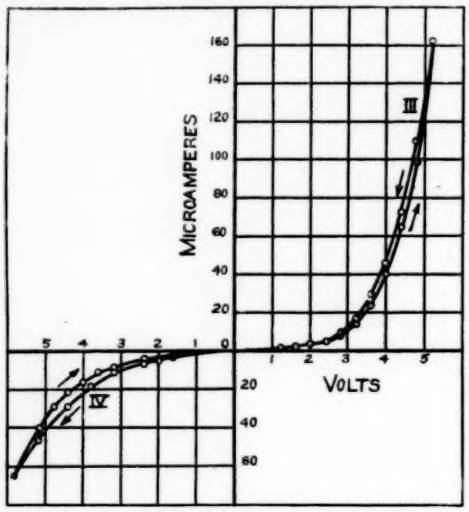

Fig. 4.

voltage was *increased* step by step did not exactly agree with the values of the current obtained with the same voltage when the voltage was decreasing step by step. The difference is indicated by the non-coincidence of the two curves marked *III* in Fig. 4. The arrows indicate the order of succession of the observations. Curve *IV* shows the corresponding effects when the current is in the opposite direction. This effect was apparently due to a slow building up of the current, and after several reversals of the current usually disappeared. The specimen that gave the curves of Fig. 4 was under a pressure of 2 kilograms.

Effects of Pressure. — Current-voltage curves were taken with the same specimen under various pressures. A series of results are shown in the curves of Fig. 5. In taking these observations the specimen was held in a clamp with jaws insulated from one another. One of the jaws was capable of being moved forward without rotation under the action of a screw and spring. The spring was cali-

brated, so that the pressure on the carborundum could be read off on a scale. Care was taken that the specimen was subjected to a steady compression without twisting. The pressure was in the direction of the current.

In taking the set of observations shown in Fig. 5 the pressure was first made three kilograms, giving the top curve of the series. The pressure was then reduced successively to two kilograms and one kilogram. From the first quadrant of the figure it is seen that

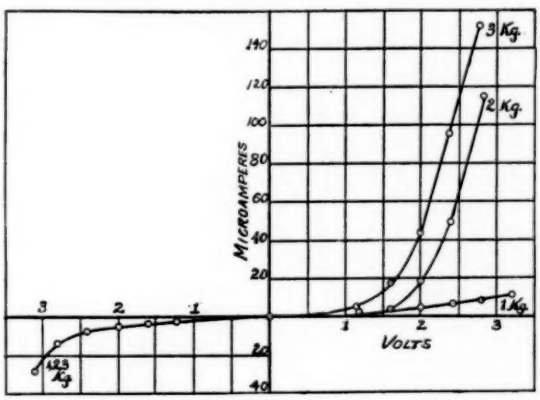

Fig. 5.

the conductivity of the specimen diminishes with diminishing pressure. This is the case when the current is in one direction. With the current in the opposite direction the current-voltage curves for the three different pressures coincide, and for the case of the pressure equal to one kilogram the prevailing conductivity is actually reversed with respect to its direction in the case of the higher pressure.

Several experiments were made with other specimens of carborundum with considerable disparity in the results, and the curves of Fig. 5 cannot be taken to represent a general occurrence.

Many of the results of the experiments here described are apparently confused by difficulties arising at the contact of the metallic electrodes with the crystal. On account of the irregularities of the surfaces of the specimens actual contact with the electrodes in general occurs at small areas. Although this contact is apparently not loose when the pressure on the clamp is several kilograms, it is

evident that the carborundum, which is an exceedingly hard substance, will imbed itself in the electrodes to a depth depending on the pressure, and one cannot be certain that a specimen will return to its original condition when the pressure is put on and taken off or when the specimen is removed from the clamp and again replaced in it. On this account the experiments are sometimes incapable of repetition unless the clamp is left undisturbed.

Experiments with Platinized Specimens of Carborundum. — In the effort to avoid these difficulties and in the effort to ascertain what part the form of contact plays in the phenomenon of unilateral conductivity in crystals, a number of specimens of carborundum were selected with opposite faces plane and very approximately parallel. The faces thus found were parallel to the natural hexagonal base of the crystal, and were in several cases apparently the natural crystal faces.

Some of the parallel-faced crystals were platinized on one or both of their smooth surfaces by the cathode discharge so as to make firm contact with the electrodes. The metallic surfaces thus obtained were in many cases optically plane, as evidenced by the fact that when used as mirrors they did not appreciably distort the image. The platinized faces were put in contact with the electrodes. The current in these cases was perpendicular to the crystal base.

Platinized on One Face Only. — Some of the specimens, platinized on one face only, gave very remarkable unilateral conductivity. A specimen, called Carborundum 11, was platinized and broken into two parts. Table II. and Table III. show results obtained with one of these parts designated 11_b. This specimen was .6 mm. thick, with area of about 1 sq. mm. One of the faces, which was optically true, was heavily platinized. The other face was somewhat rough and was without platinum. The specimen was held in a clamp with silver jaws. Careful examination showed that the rough, unplatinized face of the crystal made contact at only a few points with the electrode on that side. Four different pressures were used, 350 grams, 1 kg., 2 kg., and 3 kg. With a given voltage the current *toward* the platinized face was in each case *greater* than the current in the opposite direction.

For a pressure 350 grams, recorded in Table II., and for voltages

below 20 volts, the current from the platinized face gave no appreciable deflection of the milliammeter, of which one division was 3.92 microamperes, while the current toward the platinum was 172 microamperes at 20 volts. At 22 volts the current from the platinum was .39 microamperes, while the current in the opposite direction was 910 times as great. At 34.5 volts the current toward the platinum was 527 times the current from the platinum.

TABLE II.

Crystal 11_b. *Thickness* .6 *mm.*; *Area* 1 *sq. mm.* *Platinized on one side.*
Pressure 350 *Grams.*

Volts.	C_1, Current toward Platinum in Microamperes.	C_2, Current from Platinum in Microamperes.	C_1/C_2.
10.5	3.92		
14	11.8		
15	25.5		
17	55		
20	172		
22	355	.39	910
23	520	.60	850
25	850	1.06	800
30	1,920	3.05	630
34.5	3,100	5.90	527

Pressure 1 *Kg.*

Volts.	C_1.	C_2.	C_1/C_2.
4.5	3.92		
6	7.84		
7	19.6		
9	39.2		
10	64.0		
11	98.0		
13	168		
15	282		
16	350		
18	600		
21	1,000		
26	2,000		
30	3,000	.75	4,000
34.5	4,200	3.92	1,070

TABLE III.

Same Specimen as Table II. Pressure 2 Kg.

Volts.	C_1, Current toward Platinum in Microamperes.	C_2, Current from Platinum in Microamperes.	C_1/C_2.
4.2	3.92	.00	
5.8	7.84	3.92	2.0
7.0	19.6	4.70	4.2
8.1	39.2	9.4	4.2
9.2	78.4	19.6	4.0
10	118	39.2	3.0
11	210	71.0	2.96
12	255	121	2.10
13	380	175	2.18
14	440	260	1.72
15.8	700	350	2.0
17	1,000	420	2.38
21.2	2,000	770	2.6
23	2,500	1,000	2.5
24.5	3,000	1,500	2.65
26.2	3,500	1,360	2.58
28	4,000	1,500	2.66
31.5	5,500	1,900	2.90
34.5	6,700	2,600	2.68

Pressure 3 Kg.

Volts.	C_1.	C_2.	C_1/C_2.
3.5	3.92	.39	10
5	12.6	2.62	4.8
6	23.5	7	3.36
6.8	39.2		
8	78.5	21.5	3.55
9	118	33.2	3.55
10.2	196	78.4	3.52
12.0	314	158	2.00
13	392	240	1.60
14.2	500	392	1.30
15	600	420	1.43
17.5	1,000	700	1.43
20	1,500	1,100	1.36
21.7	2,000	1,400	1.43
24.5	3,000	1,950	1.54
30	4,500	3,300	1.37
34.5	6,700	4,400	1.56

With the pressure 1 kg., also recorded in Table II., the conductive asymmetry of the crystal was still greater, and at 30 volts the current toward the platinized face was 4,000 times the current in the opposite direction. These results for 350 grams and 1 kg. pressure are plotted in the curves of Fig. 6. The current toward

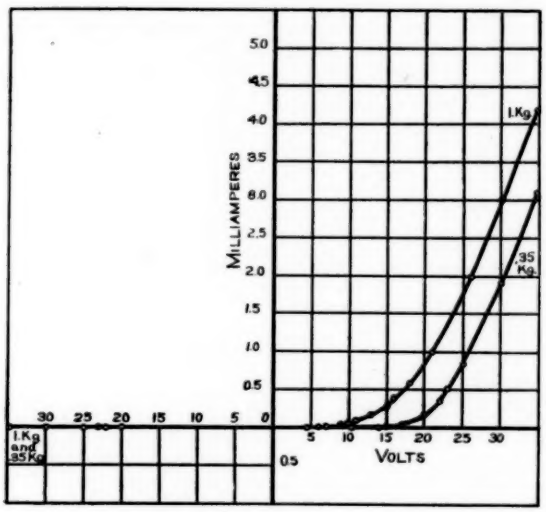

Fig. 6.

the platinized face is given in the right hand quadrant. The current in the opposite direction does not appreciably depart from the axis.

When the pressure was increased to 2 kg., and then to 3 kg. (Table III., Fig. 7) the currents in both directions were increased and the ratio of C_1/C_2 was reduced, so that the current toward the platinum was only two or three times as great as the current in the opposite direction for a given voltage.

The ratio of the current in one direction to the current in the opposite direction is reduced by increasing the pressure on the specimen from 1 kg. to 2 kg. On the other hand, if we examine these two currents with respect to their difference for a given electromotive force, it is seen that the *excess* of one of the currents over the opposite current is increased with increase of pressure.

Another small specimen also broken from crystal 11 platinized on one side gave analogous results. This small piece called 11_a had

a very sharp point at one end and a wider base at the other end. This specimen was only a fine sliver in the form of a pyramid perhaps .5 mm. long, and .2 mm. wide at the base. The platinized side constituted the base. The pressure on it was very great for so slender a specimen, perhaps 3 kilograms, so that the sharp end was

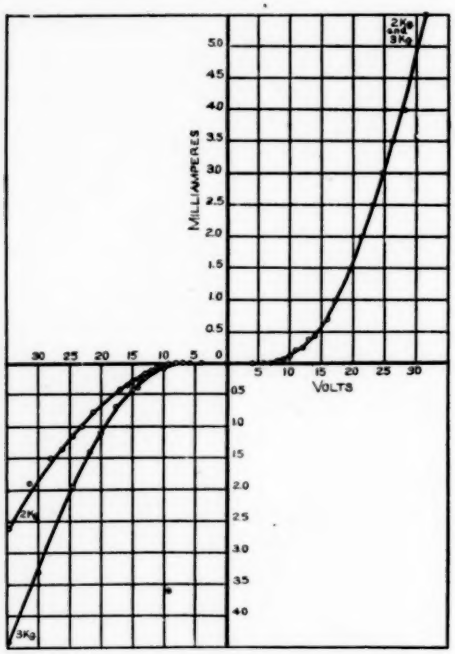

Fig. 7.

driven well into the electrode. The current toward the platinum (base) was again greater than the current in the opposite direction. The ratio of the current C_1 to C_2 in this case was 524 at 18 volts, and 271 and 32 volts.

Carborundum Platinized on Both Sides. — Specimen No. 19 gave the current-voltage values recorded in Table IV. This specimen was .82 mm. thick, had practically parallel plane faces, which were both heavily platinized by the cathode discharge. These flat faces were put into contact with the silver electrodes. The platinum of the edges, which would otherwise short-circuit the crystal, was removed by breaking away the edge of the crystal all around. This left an electrode area to the crystal of about 5 sq. mm. A

pressure of 2.5 kg. was applied to the electrodes, but the amount of this pressure so long as the contact was not evidently loose or so large as to crush the carborundum seemed to have very little effect on the resistance of the specimen.

TABLE IV.

Specimen No. 19, platinized on both sides.

Volts.	Current in 10^{-4} Amperes.		C_1/C_2.	Resistance in Ohms.	
	C_2.	C_1.		R_1.	R_2.
1	1.5	2.0	1.36	6,660	5,000
1.5	6.0	10.0	1.66	2,500	1,500
2	9.5	15	1.59	2,100	1,330
3	20	30	1.50	1,500	1,000
4	37.9	54.2	1.42	1,060	740
5	68	95	1.40	735	530
6	109	150	1.37	550	400
7	152	210	1.38	455	332
8	217	288	1.33	370	280
9	278	370	1.33	323	243
10	380	485	1.28	263	207
11	460	620	1.35	240	178
12	580	780	1.35	207	154
13	760	970	1.28	171	134
14	920	1,110	1.22	152	125
15	1,100	1,450	1.32	136	103
16	1,350	1,700	1.26	119	94
17	1,650	2,000	1.21	102	85
18	2,000	2,450	1.23	90	73
19	2,500	2,830	1.13	76	67
20	2,940	3,600	1.23	68	55
21	4,200	4,820	1.13	50	43

The ratio of C_1 to C_2 in this table in only about 1.1–1.6, but on account of the low resistance of the specimen, the *difference* between the two currents for a given voltage is large. It should be noted that the current values in Table IV. are given in 10^{-4} amperes, while in the previous Tables the microampere (10^{-6} amperes) was the unit employed. This specimen shows unilateral conductivity with currents up to one half ampere. Up to this value the current through the specimen was also remarkably steady in comparison with the unplatinized specimens. At one half ampere the crystal was considerably heated, so that the massive electrodes in contact

with the specimen became uncomfortably warm to the touch. Under this current the crystal glowed over a small area on both surfaces. Whether this glow was incandescence or a luminescence I am not able to say at present. The glow was examined with a spectroscope, but was so faint that the spectrum has not as yet been clearly made out. All of the specimens of carborundum thus far studied showed this glow at the electrodes when the current was pushed to a high value. Some specimens, apparently inhomogeneous, showed a glow also at a small region in the interior of the crystal. When the glow appeared the measurements were usually discontinued on account of the irregularity of the current readings.

Rectification of Alternating Currents by Carborundum.— In the previous experiments it has been shown that carborundum is unilaterally conductive, giving a greater current in one direction than in the opposite direction when the same electromotive force is applied in the two cases. If this property is manifested for rapid reversals of voltage, an alternating voltage ought to give more current in one direction than in the other. Experiments show this to be the case.

The form of circuit shown in Fig. 8 was designed to permit the employment of either direct or alternating voltage of the same

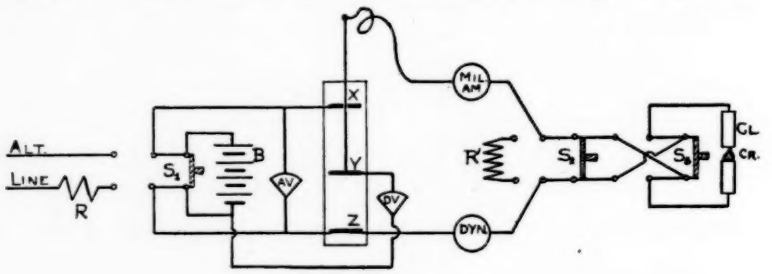

Fig. 8.

value, and to permit the ready measurement of the direct, alternating, and rectified current through the specimen.

The resistance R in the alternating line was adjusted so that the alternating current voltage between X and Z read on the Thomson alternating voltmeter AV with switch S_1 to the left was the same as the direct voltage between these plates when the switch S_1 was thrown to the right so as to put in the battery B instead of the alternating line.

Since the resistance XYZ was non-inductive it was assumed that the drop in potential between Y and Z was the same for either direct or alternating voltage. This drop between Y and Z was read, with the direct current source, on the Weston direct current voltmeter DV.

The resulting current through the crystal Cr was read on both the milliammeter and electrodynamometer in series with the crystal. With the alternating current source the former gave the rectified current and the latter gave a reading depending on the total alternating current through the specimen.

The specimen used in the experiment was taken almost at random from a beaker full of crystals of carborundum, and though this

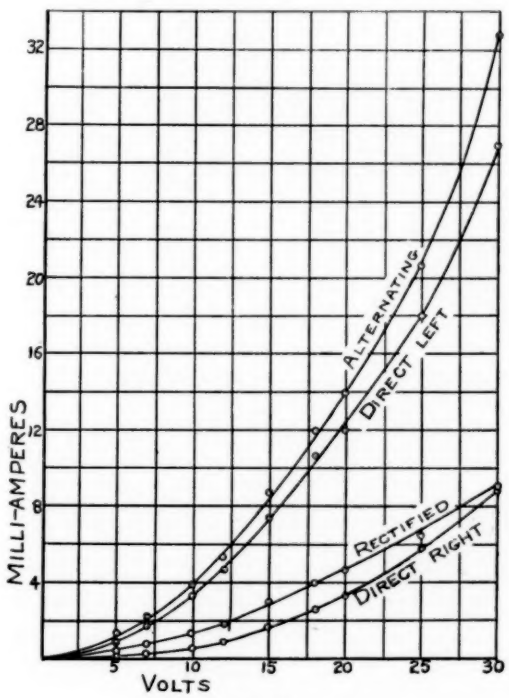

Fig. 9.

specimen does not possess polarity to the remarkable degree exhibited by specimens 11_a and 11_b, it yet shows the possibility of rectifying alternating currents by the use of carborundum. Table V. shows a series of observations which are plotted in Fig. 9. The

table contains the voltage, the direct current in the two opposite directions obtained with the battery as source, the total alternating current as read on the dynamometer, and the rectified current with the alternating source. The alternating current made 60 cycles per second.

In Table VI. one half the difference between the two opposite values of the direct current is put in for comparison with the rectified current. This quantity $\frac{1}{2}(c_1 - c_2)$ is the reading we should have for the rectified current if a steady voltage of which the direction was reversed at a uniform rate were applied to the circuit. We should not expect agreement of the current obtained in this way with the rectified current under alternating voltage, but the fact that the two

TABLE V.

Direct, Reversed, Alternating and Rectified Current for Different Voltages.

Volts.	Current in Milliamperes.			
	Direct Left, C_1.	Direct Right, C_2.	A, Alternating.	R, Rectified.
5	.9	.07	1.3	.57
7	1.7	.2	2.21	.98
10	3.22	.52	3.9	1.61
12	4.53	.89	5.22	2.08
15	7.5	1.16	8.8	3.35
18	10.7	2.53	11.7	4.13
20	12.0	3.25	14.0	4.7
25	18.0	5.6	20.7	6.25
30	26.8	8.9	32.7	9.0

TABLE VI.

Efficiency.

Volts.	Rectified.	$(C_1 - C_2)/2$.	R/A.	$\pi R/2A$.
5	.57	.41	.44	.69
7	.98	.75	.44	.69
10	1.61	1.35	.42	.65
12	2.08	1.82	.40	.63
15	3.35	3.17	.38	.60
18	4.13	4.09	.36	.56
20	4.7	4.37	.34	.52
25	6.25	6.2	.30	.47
30	9.0	8.9	.28	.44

sets of values are of the same order of magnitude gives a rough method of predicting the rectified current from the reversed direct currents.

Efficiency of Rectification. — Table VI. contains also data in regard to the efficiency of rectification by carborundum. In the fourth column is the ratio of the rectified current, read on the Weston ammeter, to the total current read on the electrodynamometer. This, however, does not give the true efficiency because the current is no longer sinusoidal, and therefore the reading of the dynamometer, which is a deflection instrument calibrated with a direct current, does not give, even approximately, the proper value of the alternating current for the comparison. If the rectification were perfect the reading of the dynamometer would have to be multiplied by $2/\pi$ to give the same reading as the direct current ammeter. The fifth column of the table contains the ratio of the rectified current to $2/\pi$ times the dynamometer reading. This also is not the true efficiency, which is, however, perhaps somewhere between the ratios of column four and those of column five.

Since the efficiency is different for different specimens it is perhaps not important to enter into a more accurate discussion in this place.

The results of Tables V. and VI. show that direct reading instruments in series with a rectifying crystal are capable when properly

Fig. 10.

calibrated of reading on alternating current voltage, and are, therefore, capable of being used in the measurement of alternating currents. There is given below a description of an alternating current voltmeter constructed in this way.

The Modification of Wave Form by a Crystal of Carborundum. — In order to find out by a direct method the effect of a unilaterally

conductive crystal on the wave form, when the crystal was used in a circuit with approximately a sinusoidal impressed voltage, photographs were taken with a Braun's cathode tube. The resulting cycle is shown in Fig. 10. For the experiment Professor Trowbridge kindly placed at my disposal his high-voltage storage battery, for use in producing the cathode beam in the tube. It was found that 20,000 volts in series with a water resistance with running water gave a brilliant and steady spot on the fluorescent screen.

Details of the apparatus are shown in Fig. 11. The coils *cc*

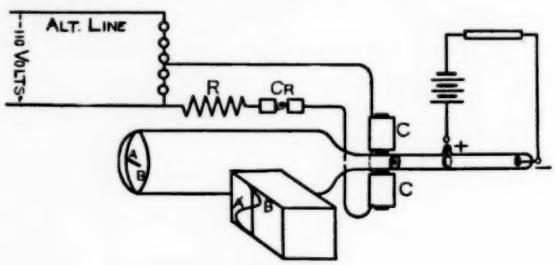

Fig. 11.

for deflecting the beam had together a resistance of about 500 ohms. These coils were provided with cores of soft iron, since it was found impossible to dispense with the iron and have sufficient deflection to show the wave form. The deflecting coils and an additional resistance of 3,720 ohms were in series with the crystal *Cr*. The alternating voltage applied to the circuit containing the crystal was 44 volts, obtained as the drop around two lamps of a lamp-resistance in circuit with the 110-volt line. The crystal was submerged in oil.

The current through the crystal *Cr* and the coils *cc* deflected the cathode beam so that the luminescent spot was spread out in a horizontal line *AB* on the fluorescent screen of the Braun tube. The deflection in one direction was greater than in the opposite direction. Viewed in a mirror revolving with axis horizontal, the fluorescent line was spread out vertically into the wavey line *A'B'*. This curve moved stroboscopically with the mirror. In order to take a photograph the speed of revolution was adjusted to synchronism with the cycle to be photographed. This was done by the adjustment of a conical bearing connecting the motor with the

mirror. The motor was run at approximately constant speed, so that when the coupling was properly adjusted, the image in the mirror could be held stationary for sufficient time to make the exposure (15 seconds.)

The cycle shown in the photograph of Fig. 10 is in form what one might expect from the current-voltage curve previously given. The horizontal line was traced by the spot when the current was shut off, and is therefore the line of zero current. This is the proper axis of the cycle. The mirror was moving at uniform speed so that distances along the axis of the cycle are proportional to the time. The approach of the curve to a horizontal where it crosses the axis is due to the fact that the apparent resistance of the crystal for low voltages is more than the resistance for high voltages. The fact that this effect is shown with the alternating voltage reversing 120 times per second indicates that the change of apparent resistance with the change of current is extremely rapid. If the effect is due to heat, the photograph shows that a marked minimum of temperature coincides in time very closely with the minimum of current. We should perhaps hardly expect the crystal in contact with massive electrodes and submerged in oil to show decided fluctuations of temperature with each fluctuation of the rapidly reversing current.

Another matter for examination in the photograph is the comparative time interval between successive minima of current; that is the comparative time intervals between the zero values of the current on the oscillogram. These intervals are apparently unequal. We cannot, however, be quite sure of this inequality, because on account of the breadth of the band, the positions of these points are somewhat uncertain in the photograph, and if we place the first and second as near together as possible and the second and third as far apart as possible consistent with the picture, the two distances become about equal. If these distances on further experiment should prove to be unequal, we should have evidence of a *persistent* electromotive force in one direction acting to displace the cycle with respect to the axis. This would mean that the rectifying effect was in part an integral effect.

Such an integral effect exists, for example, in the case of a thermo-

electromotive force produced by an electric current, which does not disappear as soon as the current is removed. Also an integral effect arises in galvanic polarization, which does not disappear as soon as the current producing it is removed. So we might expect in the case of the crystal an integral electromotive force persisting in one direction from cycle to cycle. From the picture we cannot say for certain that such a persistent electromotive force exists. If such a persistent electromotive force does exist, it does not necessarily follow that the cause is thermoelectric or galvanic in character.

Experiments on Question of Thermoelectric Origin of Electric Polarity in Carborundum. — Evidence that the action is not thermoelectric is given by the fact that submerging the crystal in oil does not appreciably change its behavior. Also, blowing air over the crystal when it is rectifying an alternating current does not change the current. When used on an alternating voltage, a lighted match held under the piece so as to heat it chiefly at one electrode does not much change the rectified current. These facts seem to be inconsistent with the assumption that the direct electromotive force obtained with an alternating voltage is thermoelectric in origin.

A direct experiment in search of an electromotive force persisting after current was removed was performed as follows. By means of a relay making 60 complete vibrations per second, a direct current from a battery of 35 volts was sent through the crystal of carborundum for intervals of about 1/120 second, disconnected and a capillary electrometer put about the crystal for alternate intervals of about 1/120 second. The capillary electrometer gave indications of a persistent electromotive force of less than .002 volt. This is entirely inadequate to account for the unilateral conductivity of the substance, which would require in some cases a back voltage as high as 10 or 15 volts to explain the phenomenon.

After a presentation of the data for other crystals now under investigation, it is proposed to return to a discussion of possible explanations of the phenomenon.

Crystal Rectifiers Employed in the Construction of Alternating Current Measuring Instruments. — On account of the unilateral conductivity of carborundum and certain other crystals to be later

referred to, it is possible by the use of such crystals to adapt an ordinary direct current measuring instrument to the measurement of alternating currents. The utility of these crystals for this purpose will depend upon their permanence and constancy when submitted to long continued use. For this purpose long runs are at present being made.

An alternating current voltmeter making use of carborundum as the rectifying substance has stood satisfactory tests for several weeks. The crystal is put in a metallic clamp enclosed in a tube containing oil, and is used with a delicate Weston milliammeter, with a scale of 100 divisions, each division being 3.92×10^{-6} amperes. Shunt and series resistances as shown in Fig. 12 are used. When these resistances are properly chosen, the scale of the instrument used as an alternating current voltmeter will be nearly uniform over a wide range, or if desired the readings for the small voltages may be spread out instead of being compressed as in the ordinary alternating voltmeters. By dispensing with some of the series resistance, or by changing the shunts it is possible to give the instrument a multiple scale.

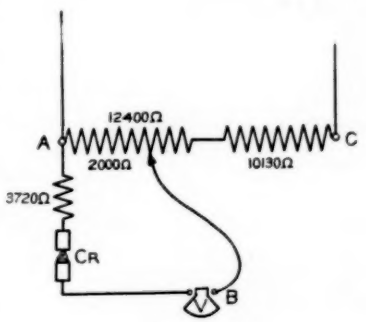

Fig. 12.

Calibration curves for the instrument are shown in Fig. 13 and Fig. 14. Curve *I*, Fig. 13, shows the reading of the direct current Weston instrument for different alternating voltages, the instruments being provided with the shunt and series resistances shown in Fig. 12. Curve *II*, Fig. 13, is the corresponding calibration curve without the 10,130 ohms series resistance. The curves are both nearly straight above 20 volts.

When the large series resistances and shunt resistances are omitted and the current sent directly through the 3,720-ohm resistance, the crystal, and the direct current millivoltmeter (resistance 638 ohms), curve *III*, Fig. 14, is obtained. The corresponding direct current curves *IV* and *V* are given in the same figure. With this arrangement the instrument may be read with accuracy from 2

to 10 volts. The deflection at 5 volts alternating is 36 one-hundredths as great as the larger *direct current* with the same voltage, and 7 times as large as the corresponding direct current in the opposite direction. That is to say the loss in sensitiveness of the

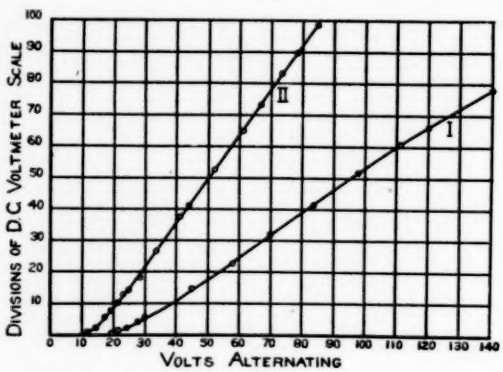

Fig. 13.

direct instrument used on the alternating voltage is not large. Of course, difficulty may in some cases arise in the use of these crystals on account of their high resistance. However, by the use of

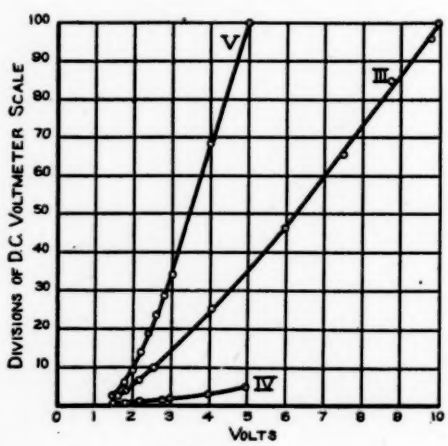

Fig. 14.

a transformer the voltage of an alternating current may be stepped up, and this difficulty avoided. Also by platinizing the specimens, the resistance of the crystal may be greatly reduced.

The alternating voltage used in this experiment made 60 complete cycles per second. Similar measurements with 175 cycles gave substantially the same curves. When used as an alternating current voltmeter, the instrument may be left permanently in circuit without heating or deterioration. The temperature coefficient of conductivity is considered on page 54 below.

The Unilateral Conductivity of Carborundum Used in the Measurement of Telephone Currents. — When it is desired to measure smaller voltages than those given above, a step-up transformer may be used to raise the voltage of the alternating circuit. On account of the high resistance of the carborundum the factor of transformation may be very great, so that the primary of the transformer may be of such small resistance that its introduction into a circuit will not materially modify the conditions of the circuit. In a telephone line a transformer of this kind might quite accurately replace one of the transformers between the talking and listening circuits of the line.

Very feeble currents may be measured in this way, and many experiments on resonance in telephone circuits may be performed. The following experiment serves as an example.

An apparatus similar to that devised by M. Wien, consisting of a rotating wheel carrying iron plugs between the poles of a small permanent magnet, was used to produce a feeble alternating current of 1,440 cycles per second. This alternating current was sent through the primary coil of a telephone transformer. The secondary of this transformer was connected with a delicate ammeter through the crystal of carborundum No. 19, platinized on both sides. A deflection of 16 divisions was shown by the milliammeter. Now the primary circuit was broken and a condenser of variable capacity was inserted in series in the primary circuit. The deflections obtained with various values of the capacity are plotted in Fig. 15. The dotted line shows the deflection without the condenser (C = infinity). The curve shows clearly a resonant value of the capacity at .45 microfarad.

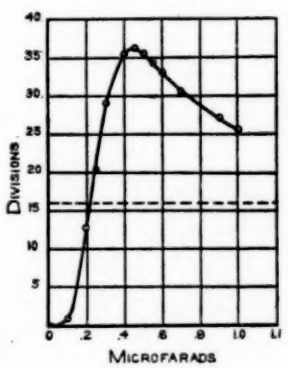

Fig. 15.

The rectifying action of carborundum is applicable in a similar manner to the study of many important problems in telephony.

I have found some other crystals that show greater sensitiveness than carborundum, when used as rectifiers of weak alternating currents. Experiments with these substances will be communicated in a subsequent paper.

Temperature Coefficient of Conductivity of Carborundum. — One of the greatest difficulties in the way of the use of carborundum in standard measuring instruments for alternating currents is the high temperature coefficient of this material. The platinized specimen No. 19 heated in an air bath gave the results shown in Fig. 16.

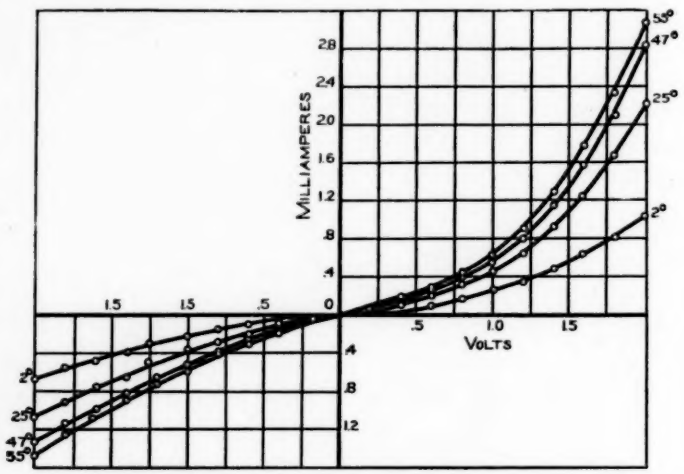

Fig. 16

Direct current was used and current-voltage curves were taken at various temperatures; namely at 2°, 25°, 47°, 55° C. By a reference to the figure it is seen that the conductivity of the substance increases with increase of temperature.

More accurate observations are shown in Fig. 17. In this case the crystal was heated in an oil bath. The electromotive force was constantly 2 volts. The curve C_1 was obtained with the current in one direction, the curve C_2, with the current in the opposite direction. The observations were carried down below 0° C., so that any moisture present on the crystal would be frozen, and would hence be evidenced by a sudden change in the inclination of the curve.

No such change was found, indicating that the presence of free water on the crystal could not account for the unilateral conductivity. It is proposed to carry the measurements to a still lower temperature at a subsequent time in the effort to see if a temperature can be found at which the conductivity of the crystal ceases to be unilateral.

At 20° the conductivity along the curve C_1 increases 2.06 per cent. per degree, while the corresponding coefficient along C_2 is 1.52 per cent. per degree. It is interesting to note that the temperature coefficient of conductivity of neutral salts in very dilute

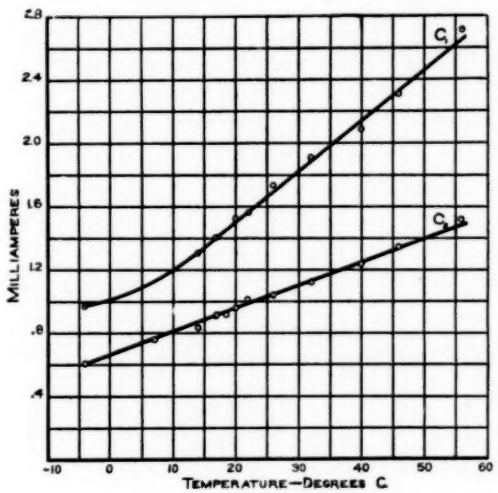

Fig. 17.

solutions is between 2.2 and 2.4 per cent. per degree, and is likewise an increase of conductivity with increase of temperature. The corresponding coefficient for copper and other pure metals is negative instead of positive and has a value in the neighborhood of .38 per cent. per degree C. It thus appears that in the matter of temperature coefficient carborundum resembles an electrolyte rather than a metallic conductor.

On the Action of Carborundum as a Detector for Electric Waves. — The facts obtained in the above experiments contribute to an explanation of the action of carborundum as a detector for electric waves. The current-voltage curve of carborundum is not linear;

the apparent resistance of the substance drops when the current is increased, so that we might anticipate that electric oscillations through the carborundum, constituting an increase of the current, would cause a decrease of resistance and consequently a sound in the telephone, provided a local battery is used in series with the detector and telephone.

Even if the carborundum were not unilaterally conductive, this drop in resistance with increase of current would characterize the substance as a detector for electric waves as may be seen by reference to Fig. 2. For example, let us suppose that the local battery has a voltage of 2 volts, and suppose that the oscillating voltage impressed by the incident waves is ½ volt. When this impressed E.M.F. is in the same direction as the local voltage the total voltage is 2.5, and the resulting current from the curve is 8.4 microamperes. When the impressed voltage is opposite to the local voltage, the total voltage is 1.5 volts, and the resulting current 1.8 microamperes. The average of these two current values 8.4 and 1.8 is 5.1, which is an increase of 1 microampere over the local current under 2 volts. That is to say, the average effect of a train of waves produces an increase of current in the telephone or other current operated device in series with the specimen and a local battery.

This is, however, evidently not a complete explanation of the action of the carborundum; for as General Dunwoody points out in his patent application carborundum may be used as a detector for electric waves *without a local battery in circuit.*

When employed without the local battery, the action of the carborundum as a detector is undoubtedly due to its unilateral conductivity. My experiments show that under the action of an alternating voltage more current passes in one direction than in the opposite direction. In using the detector without local battery the carborundum is shunted with a telephone, and in the case of the simplest form of receiving circuit the carborundum is in series with the antenna and ground. Electric oscillations with the voltage in one direction give a large current through the carborundum, charging the antenna. When the voltage reverses the current from the antenna to ground through the carborundum is smaller, thus leav-

ing the antenna charged with a small quantity of electricity. The effect of the whole train of waves is additive, so that this charge on the antenna is cumulative. The accumulated charge on the antenna escapes through the telephone shunted about the carborundum, causing the diaphragm to move. Each subsequent train of waves causes a similar motion of the diaphragm, which is evidenced as a note in the telephone with the train frequency of the waves.

This explanation of the action of carborundum when used as a detector for electric waves brings it into agreement in essential characteristics with a number of other detectors for electric waves, which have been comparatively studied by H. Brandes.[1] This author found that "in general, conductors or combinations of conductors which do not follow Ohm's Law are capable of acting as detectors of electric oscillations, owing to their rectifying effect. A verdict as to whether such an arrangement is more or less suitable for this purpose can be derived from the continuous-current characteristic (voltage current curve). If this characteristic be symmetrical in the first and third quadrants, such an arrangement can be used as a detector, the oscillations being superposed on a suitable continuous current. If the characteristic be asymmetric, the arrangement can be used as a detector without an external electromotive force, although a suitable auxiliary electromotive force, on which the oscillations are superposed, often increases the effect. These conclusions are arrived at as a result of considering the action of Braun's pyrolusite, Holtz's valve cell, the electrolytic detector, the Wehnelt oxycathode tube, the vacuum tube, the conductivity of flames, and Ferrie's so-called electrolytic detector without polarizing E.M.F." Among the detectors here enumerated by Brandes, pyrolusite is a crystalline solid substance of the formula MnO_2.

The question whether or not the action of carborundum when used as a detector for electric waves is a heat effect goes back to the question whether the unilateral conductivity of the substance is of thermal origin. I am at present of the opinion that heat is practically a negligible factor in the process.

The following historical sketch of previous experiments on the

[1] Elektrotechn. Zeitschr., 27, pp. 1015–1017, November 1, 1906. Quotation from Science Abstracts, 2078, December 28, 1906.

conductivity of crystals and allied substances is interesting in this connection.

Historical Sketch. — In 1874 Ferdinand Braun[1] in a research on the conductivity of metallic sulphides found that the current was different in different directions, and that the resistance depended on the intensity of the current. In some cases the current in one direction was as much as twice as great as the current in the opposite direction. The substances investigated were copper pyrites, iron pyrites, galena, and copper antimony sulphide. Contact was made by the use of a clamp. He says that he found no thermo-electric effect or polarization that was even remotely explicable of the phenomenon. In discussion of his results Braun puts forth a possible explanation, based on hypotheses as to the nature of the structure of the crystals.

In 1877[2] and 1878[3] Braun returned to the subject and reported the results of further experiments, including a study of the conductivity of Psilomelan, which is a "combination of a base with manganese superoxide and water." In these experiments he showed that the pressure of the surrounding atmosphere had no effect on the phenomenon. To ascertain whether the cause of the variation of the resistance with the strength of current was due to chemical processes, he connected a copper voltameter in series with a piece of copper pyrites between silver electrodes and a piece of Psilomelan between gold electrodes. After the current from eight Grove cells had been running for nine hours, he found that the current had separated out 1.404 grams of metallic copper in the voltameter while the silver electrodes in contact with the pyrites showed no trace of silver sulphide. Also the pyrites and Psilomelan showed no appreciable decrease in weight. As further evidence that there was no electrolysis of the substances, Braun found that two electrodes of different metals at the same temperature when placed in contact with the mineral gave no current.

Another important result obtained by Braun is the fact that when a constant current is sent through the body simultaneously with an induced current from an induction coil, the resistance is lowered, not only for the former but also for the latter.

[1] Pogg. Ann., 153, p. 556, 1874.
[2] W. A., 1, p. 95, 1877.
[3] W. A., 4, p. 476, 1878.

This work of Braun was considerably discussed at the time by writers who were unable to obtain the phenomenon or who advanced the opinion that the effects were of thermoelectric or electrolytic origin. In his papers of 1877 and 1878 and in 1883[1] Braun replied to these objections, and maintained that thermoelectric processes and electrolytic polarization could not account for the phenomenon.

In 1900 and 1902 in the course of a research on the electric conductivity of compressed powders, F. Streintz[2] investigated among other things a great many oxides and sulphides of metals. None of the substances examined (in the form of finely divided powder) "showed electrolytic conductivity." Streintz also determined the temperature coefficients of conductivity, and found that compounds that at ordinary temperature conduct well have positive temperature coefficients of resistance, less, however, than the coefficients of pure metals. Compounds, on the other hand, whose conductivity at ordinary temperature is relatively small were greatly influenced by temperature, their conductivity increasing with increase of temperature. In this lattes class belong PbS, HgS, Ag_2S, and MnO_2. A very remarkable change of conductivity with temperature was obtained with Ag_2S, as has been previously noticed by Faraday[3] and Hittorf.[4]

On the question of the thermoelectromotive force of metallic couples a very interesting series of results have been obtained by A. Abt[5] in a research entitled "Thermoelectromotive Force of Some Metal Oxides and Metal Sulphides in Combination with One Another and with Simple Metals with 100° Difference of Temperature of the Contact Points." According to these experiments oxides and sulphides of metals arrange themselves in the following thermoelectric series:

Chalcopyrites	Copper	Iron
Pyrolusite	Cadmium	Pyrrhotite
Bismuth	"Nickelerz"	Antimony
Zinc	Arc Light Carbon	Pyrite
Nickel.		

Relative to the couple bismuth-antimony considered as unity the chalcopyrite-pyrite couple has a thermoelectromotive force of 10.8.

[1] Wied. Ann., 19, p. 340, 1883.
[2] Ann. d. Phys., 3, p. 1, 1900; also 9, p. 854, 1902.
[3] Faraday, Pogg. Ann., 31, p. 241, 1834.
[4] Hittorf, Pogg. Ann., 84, p. 20, 1851.
[5] Abt, Ann. d. Phys., 2, p. 266, 1900.

From this work it is evident that the thermoelectric effects with crystals may be large in comparison with metallic couples, and this fact must be taken into account in dealing with the unilateral conductivity of crystalline substances.

SUMMARY OF RESULTS.

1. Current voltage curves for carborundum are shown.

2. Carborundum is unilaterally conductive. With one specimen under 10 volts the current in one direction is 100 times the current in the opposite direction. With another specimen platinized on one side the current at 34.5 volts is 527 times as great as the current in the opposite direction under the same voltage. In another case at 30 volts the current in one direction is 4,000 times the current in the opposite direction under the same voltage.

3. As the current increases the efficiency of rectification decreases.

4. A specimen platinized on both sides has a smaller efficiency of rectification and a much lower resistance than a piece not platinized. Though the efficiency of rectification is less with the platinized specimen, the *excess* of one current over the other for a given voltage is much greater with the platinized specimen on account of its low resistance.

5. An oscillogram is given showing the distortion of an alternating current wave by carborundum.

6. A method is shown of employing crystal rectifiers in the construction of alternating current measuring instruments.

7. These instruments are applicable to the measurement of telephonic currents, and may be used in experiments on resonance in telephone circuits.

8. A determination of the temperature coefficient of conductivity of carborundum is given. This coefficient is in the neighborhood of the temperature coefficient of weak electrolytes.

9. A discussion is given of the action of carborundum in General Dunwoody's detector for electric waves.

10. No theory is given as to the cause of the unilateral conductivity of carborundum. A number of other crystals have the same property, and it is proposed to accumulate data in regard to these other substances before attempting an explanation.

JEFFERSON PHYSICAL LABORATORY,
HARVARD UNIVERSITY, CAMBRIDGE, MASS.,
May 6, 1907.

THEORY OF CONTROL OF THE ALTERNATING CURRENT GALVANOMETER.

By A. H. Taylor.

THE instrument to be discussed in this paper is really a shunt circuit dynamometer with a laminated iron field. Or, it is an ordinary D'Arsonval having a laminated iron field separately excited by a current drawn from the same source as the alternating current to be detected.

Since the moving coil hangs in an alternating field, it must either not be wound on a copper frame, or else the reaction of the frame (due to eddy currents) must be compensated for after the manner of Stroud and Oates,[1] who used this method of controlling the initial equilibrium position of the moving coil. A more satisfactory method of control is the one devised by Terry;[2] a method which allows equally flexible control on a great variety of external circuits.

Providing that the suspensions are not too heavy, they have in general but little to do with the initial position of the coil when the field current is on, since the induced currents in the coil react with the field in such a way as to produce a torque whose direction depends on whether the induced current leads or lags with respect to the induced E.M.F. Reference to Fig. 1 will explain this. Let AA be a fixed line, while OF is a rotating vector whose projections on AA give the instantaneous values of the field. This alternating field will induce in the coil the E.M.F. represented by the rotating vector OE. Since the coil has a large inductance (80 millihenries), the induced current will generally lag, and may be represented by the vector OC. But if the outside circuit contain sufficient capacity (in series or in shunt) the induced current may lead the E.M.F., and hence must be represented by the vector OC'. Since the torque will be proportional to the product of field and induced current, it is evident that the torque due to OF and OC

[1] Stroud and Oates, Phil. Mag., Dec., 1903.

[2] Terry, Phys. Rev., Sept., 1905.

will be opposite in direction to that due to OF and OC', while if there is no lag or lead, there will be no torque except that of the suspension.

The fact that the projections of OF and of OC during the greater part of the cycle have opposite algebraic signs, means that the relation shown in Fig. 2 holds for most of the cycle. N and S repre-

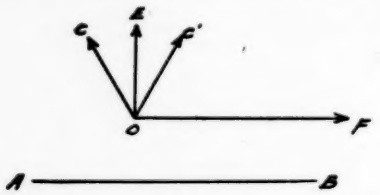

Fig. 1.

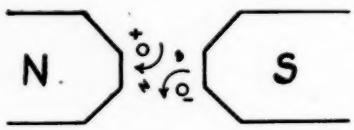

Fig. 2.

sent the instantaneous polarity of the pole pieces of the field; n and s represent the instantaneous polarity of the coil faces.

The reaction will evidently be such as to hold the coil in a position of stable equilibrium with its plane parallel to the field.

The direction of torque may be determined analytically as follows: Let the magnetic field be $f = F \cdot \sin \omega t$, where F is the maximum value. The induced E.M.F. is

$$E = -\frac{dn}{dt} = -K \cdot \frac{df}{dt}$$

where K is a constant depending on the geometry of the coil, and the angle its plane makes with the field. The induced E.M.F. may be written $e = -E \cos \omega t$ where E is the maximum value. If Z is the impedance of coil and circuit, the induced current is

$$i = -\frac{E}{Z} \cos(\omega t + \varphi) = -I \cos(\omega t + \varphi)$$

when I is the maximum value, and φ is the phase angle. The torque is

$$T = \int_{2\pi/\omega}^{0} ifC \cdot dt = \frac{CFI}{2}(. \sin \varphi)$$

where C is a constant depending on the magnetic moment of the coil. If the induced current lags, φ is negative and sin φ is also negative. Hence the torque is negative. If the induced current leads, φ is positive, and hence the torque also is positive.

To use the galvanometer as a detector of alternating current, it must be permanently connected to the circuit. If the coil is too stable in the first position, with its plane parallel to the field, it must be shunted with a capacity whose effect may be regulated by a resistance in series with the same; if the coil turns its plane perpendicular to the field, it must be shunted with a resistance. Hence, on any but a very low resistance external circuit, the control circuit may be regulated very easily so that the coil is not constrained by its own induced currents. Then the current to be detected is applied to the external circuit, and produces its effect as in any galvanometer.

A current cannot be *measured* unless the relative phase angle between current and field be known, but for bridge work the instrument is extremely useful, being fully as sensitive as any instrument for the detection of alternating currents, and possessing the great advantage of being applicable to commercial circuits of ragged wave-form and inconstant frequency. It is not possible to obtain a current through the coil 90° out of phase with the field under any ordinary conditions of bridge work; if it were momentarily possible, it could last for only a very short time on account of the variations in frequency of the average commercial circuit. Hence there will, in general, always be a deflection when a current passes through the coil.

This galvanometer has been used for precise A.C. bridge work in this laboratory for three years, giving excellent results in the hands of students, and as many inquiries concerning it have come to me from those compelled to use commercial circuits for A.C. precision work, I am taking this opportunity of pointing out its merits and demerits.

PHYSICAL LABORATORY,
UNIVERSITY OF WISCONSIN.

THE EXTRA-TRANSMISSION OF ELECTRIC WAVES.

BY W. B. CARTMEL.

LAST Christmas I presented at the New York meeting of the American Association for the Advancement of Science a paper, having for its title "The Optical Analogue of Certain Electrical Experiments." In it I tried to account for a phenomenon observed by Messrs. Blake and Fountain,[1] and mentioned in a paper they have recently published, in an entirely different way from what they did. Since then a paper by Mr. Schaefer[2] has appeared, in which he tries to show that it is unlikely that the phenomenon exists at all. For this reason I have undertaken in the present paper to discuss the experiments of Blake and Fountain from a somewhat different standpoint than theirs. It seems to me that the results obtained by these two experimenters are not only not unreasonable, but they are precisely what one ought to have expected.

In their experiments Blake and Fountain allowed electric waves to fall upon sheets of glass upon which tin-foil strips had been pasted, and also upon sheets of bare glass, and found that when the tin-foil covered sheets of glass were used, under some circumstances there was less energy reflected, and more transmitted than was the case with bare glass. This they account for by assuming that glass covered with tin-foil strips constitutes a medium having a different index of refraction than that of glass. To me it seems more plausible to suppose that since the total energy reflected by bare glass is the vector sum of the amounts reflected by the front and back surfaces of the glass, and since the change of phase produced at a surface on which there are tin-foil strips is different from that produced at a bare glass surface, it might well happen that the tin-foil strips would cause the difference of phase between the energy reflected at the two surfaces to be more favorable to destructive in-

[1] F. C. Blake and C. R. Fountain, PHYS. REV., XXIII., p. 257, 1906.
[2] C. Schaefer, PHYS. REV., XXIV., p. 421, 1907.

terference, and that there would be a consequent diminution in the reflected intensity and an increase in the transmitted. That such a change of phase is produced by screens of resonators is more than likely, especially in view of the fact that gratings produce a change of phase, as has recently been shown experimentally by Messrs. Schaefer and Laugwitz.[1]

If the above explanation of the phenomenon of extra-transmission be correct, it ought to be possible to obtain the same effect with ordinary light. All that is necessary is a sufficiently thin film of one substance and upon this a very much thinner film of some other substance, to produce the necessary phase change. Now the phase change produced at the surface of a transparent substance is either zero or π while that produced by sufficiently opaque substances like the metals or certain dyes is different from either zero or π. The phase change produced by a thin film of metal depends upon its thickness as well as upon the optical contents of the metal. As the thickness of a thin film of metal diminishes the change of phase and the intensity of the reflected light will both diminish, but while with diminishing thickness the intensity approaches zero as its limit, the change of phase approaches a certain finite constant value, which is practically the same for all thicknesses less than one thousandth of a wave-length. This value is in the case of silver and gold about six tenths of the value of the phase change produced by thick plates, as has been shown by G. T. Walker[2] and by Maclaurin.[3] One ought therefore to be able to realize this effect of extra-transmission by taking a film of some substance thin enough to show interference colors, and depositing upon it a mere suspicion of silver. This coating of silver would produce the necessary phase change, while being perfectly transparent. I tried to obtain the effect by silvering pieces of blown glass which showed interference colors, but did not meet with much success. This was no doubt due to the low reflectivity of the glass which is only about 4 per cent. With substances like selenium or the aniline dyes whose reflectivity is as high as 25 per cent. or 30 per cent. for some colors, one might

[1] C. Schaefer and M. Laugwitz, Ann. der Physik., XXI., p. 587, 1906.
[2] G. T. Walker, Ann. der Physik, X., p. 194, 1903.
[3] R. C. Maclaurin, Proc. Roy. Soc., 1906. Series A, Vol. LXXVIII, p. 302.

obtain much more noticeable effects. I therefore silvered lightly a film of selenium which had been deposited on glass by cathode discharge. The result was that the film of selenium which was previously of an orange color, changed to purple. This would show that there was less purple light reflected after silvering and hence more transmitted. The film of silver was very thin — it had only required fifteen seconds to deposit it by cathode discharge — and where it fell on bare glass it was impossible to detect it by the eye. Part of the plate had been protected from cathode discharge, and the only way in which it was possible to tell where the silver had fallen was by the difference in color of the two parts of the selenium film. In another case a film of selenium too thin to show interference colors was deposited upon a glass plate and this gave the glass a light brown color. Upon part of it an extremely thin coating of silver was deposited. By transmitted light the unsilvered part appeared darker than the silvered.

Experiments were also tried with compound films of fuchsine and silver. Half of one side of a glass plate was lightly silvered with the intention of afterwards depositing fuchsine upon it. I was unable to tell which side was silvered try as I might, but I thought I remembered that I had exposed to cathode discharge the side of the glass plate which was least scratched, and so proceeded to deposit from an alcoholic solution a film of dye upon what was thought to be the silvered side. The dye showed plainly that this side had been silvered, because where the dye film crossed the silver film, it was plainly more transparent than it was where it covered bare glass. The increase in transparency was still more marked when the silver was deposited on the fuchsine instead of being between the fuchsine and the glass. Indeed a thin wash of fuchsine on glass vanished from sight almost entirely by being silvered. This I do not believe to have been due to an annihilation of the dye caused by the bombardment by the silver cathode particles because the dye reappeared when the plate was washed with alcohol: that is to say, the alcohol became colored with the dye that was on the plate.

In the discussion of my paper at the meeting of the American Association, it was suggested that possibly the films had been

chemically acted upon by the silver so as to produce a change of color or an increase of transparency. That this was not so in the case of selenium at least, was proved by a wedge shaped deposit of selenium, which showed interference colors, something like Newton's rings. On lightly silvering a part of this the interference bands of the silvered part were displaced relatively to those of the unsilvered part.

Let us now return to the experiments of Blake and Fountain. They allowed not light, but electric waves of wave-length 9.9 cm. to fall on glass plates about a meter square. One of the plates was 5 mm. thick and the others were 3 mm. thick. They found that the thick plate reflected 38 per cent. and transmitted 60 per cent. of the incident radiation, while those of 3 mm. thickness reflected 17 per cent. and transmitted 80 per cent. The fact that the sum of the reflected and transmitted radiation is nearly unity in both cases, is a very good check on the work. Furthermore, from their data we can compute the index of refraction of the glass by Airy's formula.

$$I = \frac{4a^2b^2 \sin^2 \frac{1}{2}\delta}{(1-b^2)^2 + 4b^2 \sin^2 \frac{1}{2}\delta}$$

in which a^2 is the intensity of the incident radiation; b^2 is the reflection coefficient, which is connected with the index of refraction as follows:

$$b^2 = \left(\frac{n-1}{n+1}\right)^2.$$

$$\delta = \frac{4\pi ne \cos r}{\lambda},$$

e is the thickness of the plate, λ the wave-length in air, and r the angle of refraction.

Using these formulæ one gets the same value for the index of refraction of the thin pieces and the thick piece of glass, viz.: 2.41, which would make the dielectric constant about 5.8. These results are evident from the curves shown in Figs. 8 and 9[1] of their paper.

In the curve of Fig. 8, one may notice that the addition of the resonators causes the reflected intensity to increase. This increase

[1] Ibid., p. 269.

becomes greater as the resonators are made longer, until a resonator length of 2.7 cm. is reached after which increasing the resonator length diminishes the reflected intensity. This is not because the energy reflected at the tin-foil covered surface is a maximum for a resonator length of 2.7 cm., but because the vector sum of the energy of the rays from the front and back surfaces of the plate is a maximum at this point. If the glass had been left bare and the phase relations between the rays from the front and back surfaces had been changed by varying the thickness of the plate, the maximum of the reflected intensity would have occurred at a thickness

$$t = \frac{\lambda}{2\pi n \cos r} = .65 \text{ cm.}$$

and this maximum would not have been greater than about 49 per cent. With the resonators on the plate the maximum reflected intensity was 53 per cent., which goes to show that the resonators increased the reflectivity of the surface upon which they were pasted, as well as changing the phase of the reflected energy, and the figure also shows that they introduced some real absorption, as might have been expected.

In Fig. 8 one sees that the intensity of the reflected energy never reaches the highest possible value, but attains its maximum at a resonator length of 3 cm. where the phase change is a maximum. The intensity having its maximum at the same point as the maximum phase change, causes the curve to be rather symmetrical about a center line, which is not the case in Fig. 9.

All of the foregoing considerations, it seems to me, tend to make the results published by Blake and Fountain very plausible, though I do not think this kind of reasoning should be pushed too far. There are other effects coming in due to the fact that the thickness of the glass is of the same order of magnitude as the resonator widths and lengths. The different absorptions as shown by the dotted lines of Figs. 8 and 9, and no doubt the phase changes, are also affected to some extent by the nearness of the second surface of the glass. If, however, Mr. Schaefer had only given in his paper the results which he obtained when he tried to verify Blake and Fountain's experiments, his evidence would have had more weight.

We do not know whether his glass was thick or thin, whether it had a high or low index of refraction, nor what the reflection and transmission of the bare glass was. From his curves of reflection and transmission for different lengths of resonators, together with the same data for bare glass, it ought to be possible to decide whether or not an increased transmission should be expected in his case. In obtaining an increased transmission of light through thin films by silvering them, I had no success for a long while, because I silvered the films a little too much, so that what was gained by phase change, was lost by the absorption of the silver film. Some such thing may have prevented Mr. Schaefer from obtaining increased transmission, or it may be that the index of refaction of the glass plates that he used was too low. The index of refraction of the glass Blake and Fountain used was high — the reflection coefficient was 17.1 per cent. — while some glass is known to have a very low dielectric constant, and no doubt has a correspondingly low reflection coefficient. If the dielectric constant of Mr. Schaefer's glass was 3, a by no means improbable value, the index of refraction would probably be 1.73 and this would give 7.2 per cent. for the reflection coefficient. It is evident that with such a low reflectivity the effect of extra-transmission would be difficult to observe.

Finally I wish to say that I am not in a position to judge as to the reasonableness of the criticisms Mr. Schaefer makes of the adjustments of Blake and Fountain's apparatus. Their results seem to be good. Even if Mr. Schaefer can show that Blake and Fountain's results are wrong, which I doubt very much, it ought still to be possible for some one to obtain a true extra-transmission.

JEFFERSON PHYSICAL LABORATORY,
HARVARD UNIVERSITY,
May 27, 1907.

PROCEEDINGS

OF THE

AMERICAN PHYSICAL SOCIETY.

THE RELATION BETWEEN LUMINOSITY AND ELECTRICAL CONDUCTIVITY OF FLAMES.[1]

BY F. L. TUFTS.

THE paper was in part a reply to an article by C. Fredenhagen (Annalen der Physik, Band 20, Heft 1, 1906, p. 151) in which he calls in question a conclusion of the author in an earlier paper (Physikalische Zeitschrift, 5 Jahrgang, No. 6, p. 157), to the effect that the electrical conductivity imparted to the bunsen flame by the salts of sodium and lithium respectively varied directly as the luminosity of the yellow sodium line or the red lithium line.

The author pointed out that in the experiments by Fredenhagen the real quantities compared were the resistances at the surface of his cathode and not the conductivities of the flame gases between his electrodes. By means of data already published (PHYSICAL REVIEW, Vol. XXII., No. 4, April, 1906, p. 193) and some recent experiments by the author it was shown that in the case of the red lithium line and the yellow sodium line the luminosity varies directly as the electrical conductivity imparted to the flame by the salt vapors, whether the variation is produced by varying the rate of supply of air, gas, or salt spray to the flame, or by the introduction into the flame of chlorine gas or chloroform vapors. A discussion of these results led to the conclusion that both the luminosity and the ionization imparted to the bunsen flame by the salts of lithium and sodium were directly connected with the formation in the flame of the oxides of these metals.

Some experiments were also described on the effect of adding chlorine gas to the pure bunsen flame, and it was shown that while the addition of the gas caused a marked decrease in the electrical conductivity of the flame, it produced an increase in the luminosity of the Swan spectrum. It is evident in this case that the reactions to which the Swan spectrum luminosity is due are productive of little if any of the flame conductivity while the reactions producing the flame ions are not productive of much luminosity.

[1] Abstract of a paper presented at the New York meeting of the Physical Society, March 2, 1907.

The Effect of Frequency upon the Capacity of Absolute Condensers.[1]

By J. G. Coffin.

IT has been proposed to construct an absolute condenser of two plates of optically plane glass. In order to render the faces conducting and at the same time employ the well known method of Perot and Fabry for determining their parallelism and their distance apart, it is necessary to half-silver the opposing faces of these plates. If such a condenser is to be charged and discharged many times a second as is usually the case in their comparison with others, a doubt arises as to the completeness with which they acquire their charges, in view of the inevitable fact that this conducting film has a very high resistance. A theoretical calculation was made of this effect with rather surprising results: for example if the condenser be two plates of rectangular shape 10 cm. long, resistance of 5000 ohms per lin. cm. and capacity of 100 cm. per lin. cm., it was found that at a frequency of 1000 alternations per sec. the ordinarily assumed capacity was 1 per cent. too large, while if the frequency rose to 10000 per sec. it was 65 per cent. too large.

The mathematical method employed is one involving the imaginary variable, and is somewhat similar to the method used in calculations of the influence of frequency upon the self-inductance of coils, with which the writer has had a great deal to do lately.

The two additional cases of a circular plate condenser, involving Bessel's functions, and that of a circular ring condenser involving ordinary functions, have also been solved. The results are in agreement with those for the rectangular condenser mentioned above.

In conclusion it is found that condensers with plates of high resistance when used with alternating currents cannot be assumed to have their maximum capacity as ordinarily calculated. Their effective capacity decreases with increasing frequency and this diminution may attain large values even under moderate frequencies.

On the Variations of Resistances with Atmospheric Humidity.[2]

By E. B. Rosa and H. D. Babcock.

IN the course of an extended investigation on the ratio of the electromagnetic to the electrostatic units[3] it was found that all the resist-

[1] Abstract of a paper presented at the New York meeting of the Physical Society, March 2, 1907.

[2] Abstract of a paper presented at the Washington meeting of the Physical Society, April 1 –20, 1907.

[3] By Rosa and Dorsey.

ances employed had a higher value at the same temperature in summer than in winter, the change being a gradual drift upward from early spring to midsummer, followed by a steady drift back to the same minimum in the winter. The amplitude of the change was of course small, varying from 15 to 25 parts in 100,000, but far too large to be neglected in precision work.

The possibility of this change being due to atmospheric humidity suggested itself, but it was not at first evident how an increase of resistance could be produced by an increase of humidity. The effect of leakage arising from moisture deposited on the coils is of course to decrease the resistance, and this effect would increase with the humidity. In order to ascertain the cause of the marked change in the opposite direction, to determine its magnitude and to find how to prevent it we took up last November a systematic study of the question.

It was very soon found that changes such as occurred during an interval of six months in the atmosphere of the laboratory could be induced in a few days by placing the resistances in a closed case in which an atmosphere of high humidity was maintained. Two such cases were employed, one being kept at a constant relative humidity of about 25 per cent., the other at higher humidities, ranging from 40 to 100 per cent., although seldom higher than 80 per cent., and usually about 60 per cent. Each case was provided with thermometers and recording hydrometers.

The resistances were measured generally to one or two parts in a million, always to within a few parts in a million, and corrections made for the small fluctuations in temperature. All measurements were referred ultimately to the standard one-ohm coils of the bureau, the change of resistance of which due to this cause is known to be very slight.

The following are some of the most important results so far obtained:

1. Shellacked coils of manganin wire, such as those contained in the Richsanstalt form of standards, and in potentiometers, wheatstone bridges and various forms of resistance boxes (as made by many different makers) increase in resistance as the humidity increases, and slowly come to a final value at any given humidity.

2. They gradually return to the original values when the humidity is reduced to its original point and maintained constant.

3. The change is greater on coils wound with fine wire, such as 1,000 and 10,000 ohm coils than with coils of 10 and 100 ohms. It is very slight or inappreciable on coils of 1 ohm and less.

4. Such coils of 1,000 ohms wound on metal spools change by as much as several hundred parts in a million when transferred from a dry atmosphere to one of 80 per cent. humidity.

5. Similar coils on wooden spools, the latter having been thoroughly coated with shellac as well as the wire and then baked, change sometimes five or ten times as much as the coils on metal spools.

6. Coils having a heavy coating of shellac may change more than those covered by a light coating.

7. Shellacked coils whether on metal or wood spools, remain constant in value when sealed airtight.

8. Shellacked coils submerged in oil change with change of humidity, but more slowly and to a less extent.

9. Calcium chloride placed in the bottom of the box of resistances keeps the air in the box more nearly at constant humidity, and so reduces the fluctuations of resistance, but will not keep the resistances constant unless the box is airtight.

10. In a saturated or very humid atmosphere the resistance may decrease and become unsteady due to leakage from moisture deposited on the coils. But this does not occur until after the resistance has gone up considerably, and seldom occurs except in a saturated atmosphere.

11. Coils that are covered by paraffine, whether they have been previously coated with shellac and baked or not, do not change in resistance as the humidity is increased. This is true whether the coils are on metal or wood spools, provided in the latter case the wood spool has been filled with paraffine.

12. The cause of the increase of resistance appears to be the stretching of the wire due to the shellac expanding as it absorbs moisture. On drying out in a dryer atmosphere the shellac and the wire embedded in it contract to their original volume and the resistance falls to its original value.

13. That shellacked coils kept submerged in pure oil change slowly with the humidity indicates that an appreciable quantity of moisture is absorbed by the oil and so gets into the shellac.

14. Paraffine does not absorb moisture and hence protects the shellac from atmospheric moisture when the shellacked coil has been dipped in hot paraffine.

15. Coils on wood spools change in resistance more than coils on metal spools because the shellacked wood spool, as well as the shellac in which the wire is embedded, expands by absorption of moisture.

16. Shellacked coils kept in air and submerged in oil for measurement show the same changes as when measured in air. The moisture in the shellac which has caused the shellac to swell and stretch the wire, comes out very slowly into the oil. If the spool is dry, it may gain moisture from the oil (see 13).

17. Sterling varnish absorbs moisture and expands, causing the resistance to increase, but not as much as shellac.

18. Shellacked coils that have been kept in a relatively dry atmosphere of 25 per cent. of humidity lose moisture and decrease in resistance when placed in a closed space with phosphorus pentoxide.

19. Metal spools covered with dry silk and wound with dry silk-covered wire absorb enough moisture in a humid atmosphere to appreciably increase their resistance.

20. Shellacked coils that have been baked increase in resistance after cooling (1) by reason of the aging of the wire, and (2) by absorption of moisture from the air.

21. The decrease in resistance of a shellacked coil due to temporary or accidental heating is partly due to the driving off of moisture by the heat.

22. High resistances of manganin coated with paraffine or sealed in an airtight space are remarkably constant in value, apparently as constant as low resistances.

23. Old resistance boxes containing manganin coils coated with paraffine show no changes due to humidity, and may even be plunged in water without appreciable changes of resistance. This shows that the paraffine does not absorb moisture and is not cracked appreciably by age.

24. Shellacked coils submerged in water absorb moisture so fast as to become polarized, and their resistance decreases rapidly instead of increasing as in a moist atmosphere.

A new form of sealed resistance standards are being developed.

BUREAU OF STANDARDS,
WASHINGTON, April 15, 1907.

THE SIMULTANEOUS MEASUREMENT OF THE CAPACITY AND POWER FACTOR OF CONDENSERS.[1]

BY FREDERICK W. GROVER.

IN a condenser having absorption, the angle of advance between the current and the electromotive force is less than 90° by a small angle θ, the power factor being equal to sin θ. A condenser having absorption is equivalent, therefore, in its effect on the phase of the current, to a capacity in series with a small resistance r of such a value that $\tan \theta = pCr$ (where $p = 2\pi$ times the frequency) or to a capacity in parallel with a large resistance R such that $\tan \theta = 1/pCR$.

The power factor of a condenser is more accurately measured by determining this small angle θ than by direct measurement of the energy loss itself. Such measurements of the phase difference have been made by Potts[2] using one of Rowland's electrodynamometer methods, and by Rosa,[3] who has developed a number of dynamometer methods, and shown that they give results in good agreement with one another, and with his results on the same condensers by a calorimetric method.[4]

[1] Abstract of a paper presented at the Washington meeting of the Physical Society, April 19-20, 1907.

[2] Amer. J. Sci. (4), 10, p. 91, 1900; Phys. Zs., 2, p. 301, 1901.

[3] Bulletin of Bureau of Standards, 1, p. 383, 1905.

[4] Rosa, PHYS. REV., Feb., 1899.

In 1891 Max Wien[1] gave several alternating current bridge methods for the measurement of inductances and capacities. Among these is one where the absorption of the condenser to be tested, which he represented by a resistance in parallel with a capacity, is compensated for by an adjustable resistance in series with the standard capacity. The absorption of the latter is assumed to be zero. An elaborate investigation of the energy losses in the dielectric of cables has been published, since the work in this paper was begun by Monacsh,[2] in which Wien's method was successfully used.

The following three bridge methods have been found satisfactory. In the first three, each side of the bridge consists as usual of a capacity in series with one of the ratio coils. The phases of the currents in the two sides of the bridge are brought into unison,

1. By an adjustable resistance in series with the standard condenser.
2. By an adjustable resistance in parallel with the standard condenser.
3. By a variable inductance in series with that ratio arm which is adjacent to the condenser to be tested.

Method I. — This is the same in principle as that of Wien described above, except that, whereas he derived the formula on the assumption that the resistance representing the absorption was in parallel with the condenser, the author has adopted the assumption that it is in series with the condenser. Physically it is, of course, immaterial which assumption is adopted, since we are only concerned with the change in the phase of the current due to the presence of the absorption.

It is more convenient in practice to introduce a series resistance ρ into each condenser arm. Then

$$\frac{C_1}{C_2} = \frac{R_2}{R_1}$$

$$\tan(\theta_1 - \theta_2) = pC_2\rho_2 - pC_1\rho_1.$$

Method II. — To avoid the necessity of working with resistances of several megohms it is best to put a resistance ρ in parallel with each of the condensers. Then

$$\frac{C_1}{C_2} = \frac{R_2}{R_1}$$

$$\tan(\theta_1 - \theta_2) = \frac{1}{pC_2\rho_2} - \frac{1}{pC_1\rho_1}.$$

This method is not so good as the other two, since the sensible electrostatic capacities, inseparable from such large resistances, must be added to the capacities of the condensers. The power factor measurement is not, however, affected by these capacities.

[1] Wied. Annalen, 44, p. 681, 1891.

[2] Inaugural Dissertation, Dantzig, 1906.

Method III. — In practice a fixed inductance L_2 is inserted in one arm and a variable inductance L_1 in the other. Then

$$\frac{C_2}{C_1} = \frac{R_1}{R_2}$$

$$\tan(\theta_1 - \theta_2) = \frac{pL_1}{R_1} - \frac{pL_2}{R_2}.$$

This method has the advantage that nothing is placed in the arms with the condensers. It is, therefore, especially adapted to the comparison of capacities as small as 0.001 mf., where in Method I. the errors due to capacity of the large resistances r, required with such small capacities, may become appreciable.

To obtain the most precise results with these three methods, the condensers should be nearly equal in capacity and power-factor, and it is best to balance each of the two capacities against that of a third auxiliary condenser. This procedure has the following advantages:

1. Errors due to electrostatic induction between the bridge and its surroundings are practically eliminated.

2. The ratio of the two capacities is proportional to the two nearly equal settings of the same resistance box, making an accurate determination of the actual values of the coils unnecessary.

The following method has been found satisfactory and is especially valuable as a check on those already given, since it is very different in principle.

Method IV. — It has been shown[1] that in the comparison of an inductance with a capacity by Anderson's method, the effect of the absorption of the condenser on the value of the inductance obtained is in all ordinary cases extremely small. The difference in the actual value of the resistance Q' of the arm containing the inductance from the value Q with direct current, due to the absorption of the condenser, may become very appreciable.

$$Q' = Q + pL\tan\theta$$

$$\therefore \tan\theta = \frac{Q' - Q}{pL} \qquad (A)$$

If settings be made with first one condenser, and then with the other

$$\tan(\theta_1 - \theta_2) = \frac{\Delta Q'}{pL}$$

where $\Delta Q'$ is the difference between the values of Q' in the two cases, and

[1] Rosa and Grover, Bulletin of Bureau of Standards, 1, p. 312, 1905.

$$\frac{C_1}{C_2} = \frac{2r_2 + P}{2r_1 + P}$$

if the ratio arms P and R are equal.

This is inherently a substitution method, and by properly choosing L so as to give a satisfactorily large value to ΔQ, good results have been obtained.

The absolute values of the power factor of a condenser follows at once from measurements with these methods, if some standard of reference is at hand. For capacities of 0.1 mf. and smaller, comparisons were made directly with air condensers of negligible leakage. The values for the larger capacities were obtained by stepping up from 0.1 mf., using the relation that the resultant power factor f of the two condensers of capacities C_1 and C_2 and of power factors f_1 and f_2 is given by

$$f = \frac{C_1 f_1 + C_2 f_2}{C_1 + C_2}$$

As a check on this process, the power factor of a 1 mf. condenser was determined by using a formula (A) of Method IV., the observed value of $(Q' - Q)$ being corrected for the effect of the known residual inductances and capacities of the arms of the bridge. The result agreed closely with that given by stepping up from the 0.1 mf. air condenser. Care must be taken in using formula (A) that the change in resistance of the coil L with the frequency is negligible, otherwise it must be determined and allowed for.

Results.

These four methods have been tested by measurements on a large number of condensers, at frequencies between 30 and 1,000 cycles (using vibration galvanometers to balance the bridge). The capacity ratios obtained by the different methods agree to a few parts in 100,000, while the differences $(\theta_1 - \theta_2)$ do not differ more than can be explained by the errors in the setting of the bridge. It has been found:

1. For good mica condensers the angles θ may be as small as 0′ 30″ and as large as 4′. Poorer mica condensers, especially if their capacity is small may give values of θ as large as 30′ or even more.

2. Good condensers with paraffined paper for a dielectric have a value of from 5′ to 20′; for poor telephone condensers angles as large as 10° have been observed!

3. For all the mica condensers examined the angle θ diminishes slowly as the frequency rises.

4. The same result was observed with all the poorer paper condensers, but with some of the better paper condensers the effect was in the opposite direction. The changes are all greater than in the case of mica condensers.

5. Above 20° the power factor increases with rise of temperature both in mica and in paper condensers, the effect being small for the former and very large for some of the poorer paper condensers.

Since the power factor depends primarily on the absorption, the leakage having but a small effect, the value of the power factor would be expected to give a relative indication of the magnitude of those effects which depend on absorption, viz: residual charges, change of capacity with the frequency, dependence of the apparent capacity on the time of charge and discharge, etc. Ballistic measurements show that this hypothesis is substantiated qualitatively and, approximately, quantitatively. A measurement of the power factor gives, therefore, valuable information as to the quality of the condenser.

These effects of changes of frequency and temperature are being more thoroughly investigated.

AN EXACT FORMULA FOR THE MUTUAL INDUCTANCE OF COAXIAL SOLENOIDS.[1]

BY LOUIS COHEN.

VARIOUS approximate formulas have been given for the calculation of the mutual inductance of solenoids, some of them give results to a higher degree of approximation than others, but in every case the degree of approximation will depend on the dimensions of the solenoids.

In many cases it is absolutely important to know to what degree of approximation we can calculate the mutual inductance by any given formula, and for this reason it is very desirable to have an exact formula by means of which we can determine the degree of approximation of the various approximate formulas.

The exact formula obtained in this paper was derived in the following manner: We start with Neumann's expression for the mutual inductance of two coaxial circular currents.

$$M_1 = \int_0^{2\pi} \int_0^{2\pi} \frac{\cos \, edsds_1}{r},$$

and then integrate over the lengths of both solenoids we get

$$M = \int_0^l \int_0^{h_1} M_1 dx dx_1 = \int_0^l \int_0^{h_1} \int_0^{2\pi} \int_0^{2\pi} \frac{\cos \, edsds_1 dx dx_1}{r}.$$

The value obtained for M is as follows:

$$M = 4\pi n n_1 (V - V_1),$$

where

[1] Abstract of a paper presented at the Washington meeting of the Physical Society, April 19–20, 1907.

$$V = -(A^2 - a^2)c\left[F\left(k, \frac{\pi}{2}\right)\{F(k',\theta) - E(k',\theta)\} - E\left(k, \frac{\pi}{2}\right)F(k',\theta)\right.$$

$$+ \frac{c^4 - (A^2 - 6Aa + a^2)c^2 - 2(A^2 - a^2)}{3\sqrt{(A+a)^2 + c^2}} F\left(k, \frac{\pi}{2}\right)$$

$$\left. + \frac{2(A^2 + a^2) - c^2}{3}\sqrt{(A+a)^2 + c^2}\, E\left(k, \frac{\pi}{2}\right) - c(A^2 - a^2)\frac{\pi}{2}\right],$$

V_1 is obtained from V by replacing c by c_1,

$$c = l + l_1 \text{ and } c_1 = l - l_1,$$

$2l$ and $2l_1$ are the respective lengths of the two solenoids. $F(k, \pi/2)$ and $E(k, \pi/2)$ are the complete elliptic integrals of the first and second kind to modulus k where

$$k^2 = \frac{4Aa}{(A+a)^2 + c^2},$$

A and a are the radii of the solenoids $F(k',\theta)$ and $E(k',\theta)$ are the incomplete elliptic integrals of the first and second kind to modulus k' and argument θ.

$$k' = 1 - k^2$$

$$\sin^2\theta = \frac{(A^2 - a^2)^2 + c^2(A-a)^2}{(A^2 - a^2)^2 + c^2(A+a)^2}.$$

The value of the self inductance of solenoids can be easily obtained from the expression giving the value of the mutual inductance by making the radii equal and the lengths equal. Putting $A = a$ and $l = l_1$, the above expression will reduce to

$$L = 4\pi n n_1\left\{\frac{c^4 + 4A^2c^2}{3\sqrt{4A^2 + c^2}} F\left(k, \frac{\pi}{2}\right) + \frac{4A^2 - c^2}{3}\sqrt{4A^2 + c^2}\, E\left(k, \frac{\pi}{2}\right) - \frac{8A^3}{3}\right\}$$

A New Method for the Absolute Measurement of Inductance.[1]

By E. B. Rosa.

WIEN has given several methods for measuring inductances in terms of resistance and time, and Anderson's method is an excellent one for measuring inductances in terms of capacity and resistance. I have found a method of using a condenser in series with one arm of an alternating current Wheatstone bridge while the inductance to be meas-

[1] Abstract of a paper presented at the Washington meeting of the Physical Society, April 19–20, 1907.

ured is in series with the opposite arm, such that when a balance is secured and the frequency of the current is accurately measured both the capacity and the inductance can be obtained in terms of resistance and time by a single setting of the bridge. This use of a condenser in series with the opposite arms differs from Wien's use of the condenser in parallel with the same arm in which the inductance is placed.

If a_1, a_2, a_3, a_4 are the impedances in the several arms of the bridge, and R_1, R_2, R_3, R_4 the corresponding resistances, L the inductance and C the capacity. we have for balance

$$a_1 a_4 = a_2 a_3$$

If the resistances are free from inductance and capacity, so that L and C are the only inductance and capacity in the bridge,

$$p^2 L^2 = \frac{R_1}{R_4}(R_2 R_3 - R_1 R_4)$$

$$p^2 C^2 = \frac{R_1}{R_4}\left(\frac{1}{R_2 R_3 - R_1 R_4}\right)$$

If, in order to eliminate small differences in the resistance and in the inductance or capacity of the two ratio arms, the two resistances R_3 and R_4 are made equal, the equations reduce to

$$p^2 L^2 = R_1(R_2 - R_1)$$

$$p^2 C^2 = \frac{R_1}{R_4^2}\left(\frac{1}{R_2 - R_1}\right)$$

In interchanging R_3 and R_4 it is of course necessary to leave the condenser unmoved. In order to do this quickly without altering the resistance of the leads a special form of bridge has been devised.

The resistances are submerged in oil and slide wires attached for close settings. The method is well adapted for very precise measurements, using of course tuned galvanometers to be free from the effects of harmonics. The complete paper will be published in the Bulletin of the Bureau of Standards.